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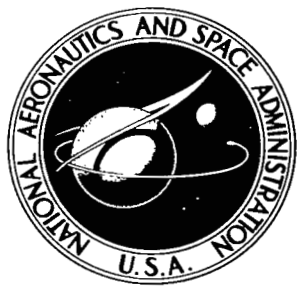

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PERFORMANCE ANALYSIS OF HIGH-ENERGY CHEMICAL STAGES FOR INTERPLANETARY MISSIONS

by Walter H. Stafford, Sam H. Harlin, and Carmen R. Catalfamo

George C. Marshall Space Flight Center
Huntsville, Alabama

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DEFINITION OF SYMBOLS

Symbol	Definition
F	Thrust, N
F/W_o	Initial thrust-to-weight ratio (based on weight at earth sea level)
f	Stage mass fraction, (W_P/W_A)
g	Gravitational acceleration, m/sec^2
H	Energy
h	Altitude, km
Δh	Altitude change, $(h - h_o)$, km
I_{sp}	Specific impulse, sec
m	Mass, kg
r	Radius, km
R	Planet radius, km
r_o	$(h_o + R)$, km
t_B	Burning time, sec
V	Velocity
V^*	Comparative velocity
V_1	Stage characteristic velocity
V_∞	Hyperbolic excess velocity
W_A	Stage weight, $(W_o - W_L)$, N
W_L	Payload weight, N
W_o	Gross weight, N
W_P	Propellant weight, N
X	Range, km
α	Angle of attack (angle between velocity vector and thrust vector), deg
ζ	Propellant mass fraction, (W_P/W_o)

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
γ	Flight path angle from vertical, deg
μ	Gravitational constant
ψ	Central angle, deg
Subscripts	
C	Burnout
esc	Escape
ex	Exhaust
f	Final
id	Ideal
o	Initial
P	Propellant
K	Circular
Abbreviations	
kg	Kilogram
km	Kilometer
m	Meter
sec	Second
N	Newton

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-1829

PERFORMANCE ANALYSIS OF HIGH-ENERGY CHEMICAL STAGES FOR INTERPLANETARY MISSIONS

By

Walter H. Stafford
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SUMMARY

The effect of earth thrust-to-weight ratios and specific impulses on trajectory parameters has been investigated for hyperbolic escape from, and braking into, an orbit about each of the planets Mars and Venus, also escape from Earth. For the escape trajectories, the thrust vector was applied tangential to the flight path and in the direction of the velocity vector. For the braking-in trajectories, the thrust vector was directed against the velocity vector. Specific impulses of 400 to 500 seconds and thrust-to-weight ratios of 0.2 to 1.0 were used.

The results of the study are presented graphically.

SECTION I. INTRODUCTION

Of fundamental importance in planning interplanetary round-trip missions is a study of the trajectory requirements. Preliminary analysis of the mission requires a rapid method of sufficient accuracy for determining the trajectory parameters. The sizing of boost vehicles is dependent, to a large extent, on the velocity requirements of the particular trajectory chosen.

The purpose of this study is to present a method for determining the trajectory parameters and vehicle mass characteristics for a specific mission, when the hyperbolic excess velocity is known.

For interplanetary missions, it is assumed that one mode of flight will be via transfer from a circular orbit about the Earth to a circular orbit about the target planet.

The approach used for escape from orbit was to determine the trajectory parameters at burnout, convert the characteristic velocity to a hyperbolic excess velocity, and then present the data graphically. The approach used for braking into orbit was to determine the arrival velocity for a given transfer trajectory and initiate burning such that circular orbit conditions are attained at burnout. The equations of motion were integrated on a RECOMP II computer, using a Runge-Kutta numerical integration procedure.

SECTION II. ASSUMPTIONS

The following is a summary of the basic assumptions used in this study:

1. For escape - Acceleration of a single stage from a reference orbit about the planet, using a constant thrust directed along the velocity vector.

2. For braking - Deceleration of a single stage from an interplanetary transfer trajectory to a reference orbit about the planet, using a constant thrust directed against the velocity vector.

3. Reference orbit about the Earth was circular with a radius of 6556.0 km.

4. Reference orbit about Mars was circular with a radius of 4000.0 km.

5. Reference orbit about Venus was circular with a radius of 6723.0 km.

6. Constant specific impulse values from 400 to 500 seconds.

7. Earth thrust-to-weight ratios were used throughout the study and were varied parametrically from 0.2 to 1.0.

8. Mean spherical Earth

$\mu_{\oplus} = 398606.6 \text{ km}^3/\text{sec}^2$

$R = 6371.27 \text{ km}$

9. Mean spherical planet Mars

$\mu_{\sigma} = 42930.0 \text{ km}^3/\text{sec}^2$

$R = 3400.0 \text{ km}$

10. Mean spherical planet Venus

$\mu_{\varphi} = 324306.2 \text{ km}^3/\text{sec}^2$

$R = 6123.0 \text{ km}$

SECTION III. ANALYSIS

For interplanetary flight, the ideal[1] total energy that must be imparted to the spacecraft is the ideal energy required to escape the gravitational field of the planet, plus the energy required to change its path about the Sun. The ideal energy required to escape the gravitational attraction of a planet can be determined from two-body mechanics to be $H_{esc} = 2\mu/r$ and the energy needed to alter the flight path about the Sun, H_{∞}, is determined by characteristics of the interplanetary trajectory.

For determining the vehicle size necessary to inject the spacecraft into the interplanetary trajectory, it is convenient to express the ideal total energy, $H = H_{esc} + H_{\infty}$, in terms of a burnout velocity. This produces equations of the following forms:

$$V_C = \sqrt{H_{esc} + H_{\infty}} \tag{1}$$

or

$$V_C = \sqrt{(V_{esc})^2 + (V_{\infty})^2} \tag{2}$$

---

(1) The term "ideal" refers to an instantaneous change of energy.

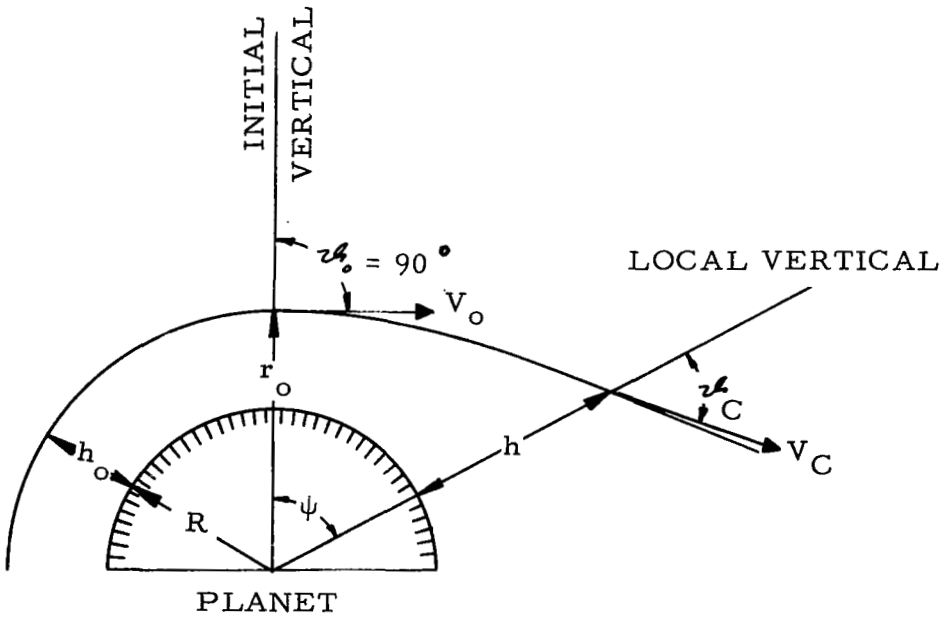
When considering finite vehicle systems there is an additional energy requirement, H_{loss}, which results from expending the propellants at different energy levels. Therefore, the total velocity increment for the injecting stage is now

$$V_1 = V_C - V_o + V_{loss} \tag{3}$$

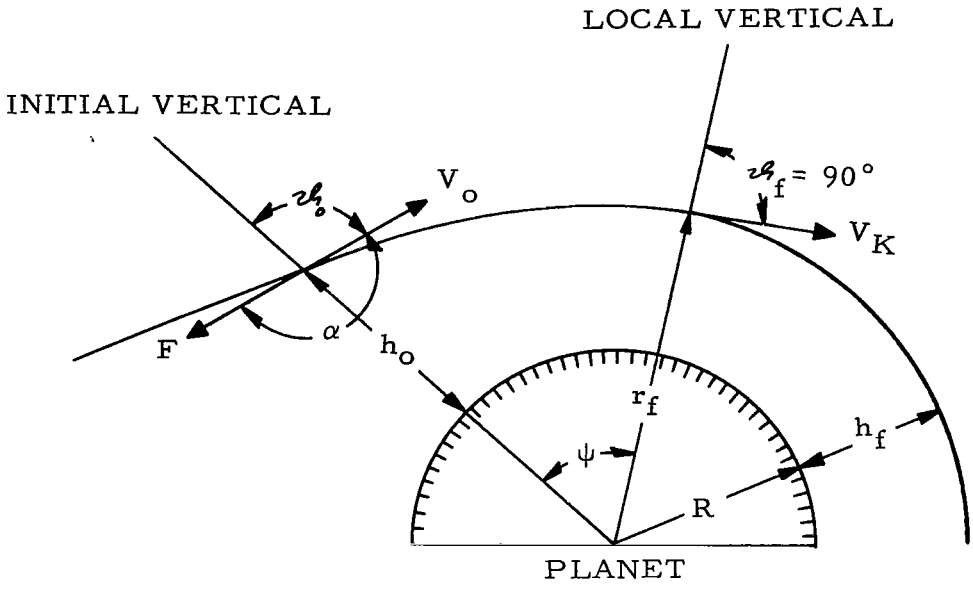
where V_o is the initial velocity. The vehicle mass characteristics can be determined from the equation

$$\frac{W_o}{W_C} = e^{\frac{V_1}{V_{ex}}} \tag{4}$$

To accomplish this study, the two-degree-of-freedom equations of motion were numerically integrated.



SKETCH a. ESCAPE FROM ORBIT



SKETCH b. BRAKE TO ORBIT

Referring to the above sketches, "a" for escaping from and "b" for braking into, computations were made for a point mass moving in a plane using the following equations of motion:

$$\dot{V} = \frac{F \cos \alpha}{m} - \frac{\mu}{r^2} \cos \gamma \tag{5}$$

$$V\dot{\gamma} = \frac{F \sin \alpha}{m} + \left(\frac{\mu}{r^2} - \frac{V^2}{r} \right) \sin \gamma \tag{6}$$

$$\dot{r} = V \cos \gamma \tag{7}$$

$$\dot{\psi} = \frac{V \sin \gamma}{r} \tag{8}$$

where

$$m = m_o + \int \dot{m}\, dt \tag{9}$$

and

$$\dot{m} = - \frac{F}{V_{ex}} \tag{10}$$

The velocity and flight path angle may be obtained by integrating the equations of motion

$$V = V_o + \int \dot{V}\, dt \tag{11}$$

$$\phi = \phi_o + \int \dot{\phi}\, dt \tag{12}$$

The range and pericenter altitude can then be calculated by the relations

$$X = X_o + \int \frac{R}{r} V \sin\phi\, dt \tag{13}$$

$$h = h_o + \int \dot{r}\, dt \tag{14}$$

and the central angle is

$$\psi = \psi_o + \int \frac{\dot{X}}{R}\, dt \tag{15}$$

The initial weight of the stage is

$$W_o = W_C + W_P \tag{16}$$

The velocity expended by the stage is the characteristic velocity, or

$$V_1 = V_{ex} \ln\left(\frac{1}{1 - \zeta}\right) \tag{17}$$

Then the velocity losses are the difference between the characteristic velocity and the change in comparative velocity, or

$$V_{loss} = V_1 - \Delta V^* \tag{18}$$

where the comparative velocity is

$$V^* = \sqrt{V^2 + 2\mu \left(\frac{1}{R} - \frac{1}{r}\right)} \tag{19}$$

For ascent from $r = r_o$ to $r = r_f$ the change in comparative velocity is

$$\Delta V^* = \sqrt{V_f^2 + 2\mu\left(\frac{1}{r_o} - \frac{1}{r_f}\right)} - V_o \qquad (20)$$

and the velocity loss due to gravity is

$$V_{loss} = V_{ex} \ln\left(\frac{1}{1-\zeta}\right) - \left[\sqrt{V_f^2 + 2\mu\left(\frac{1}{r_o} - \frac{1}{r_f}\right)} - V_o\right] \qquad (21)$$

For descent from $r = r_o$ to $r = r_f$ the change in comparative velocity is

$$\Delta V^* = \sqrt{V_o^2 + 2\mu\left(\frac{1}{r_f} - \frac{1}{r_o}\right)} - V_f \qquad (22)$$

and the velocity loss due to gravity is

$$V_{loss} = V_{ex} \ln\left(\frac{1}{1-\zeta}\right) - \left[\sqrt{V_o^2 + 2\mu\left(\frac{1}{r_f} - \frac{1}{r_o}\right)} - V_f\right] \qquad (23)$$

SECTION IV. DISCUSSION OF RESULTS

The results of this investigation are presented graphically in Figures 1 through 11 for each of the five areas considered.[2] The stage characteristic velocity, V_1, is plotted versus hyperbolic excess velocity, with earth thrust-to-weight ratios as a parameter, in Figures 1 through 5.

Figures 6 and 7 show the velocity losses, V_{loss}, due to gravity for specific impulse values of 400 seconds and 500 seconds, respectively. These losses tend to zero as the thrust-to-weight ratio is increased. The flight path angle, at burnout for departure trajectories and at initiation of burning for braking trajectories, is shown versus hyperbolic excess velocity in Figure 8.

Figure 9 gives the change in altitude. This change is the difference between the altitude of the reference orbit about the planet and the altitude at burnout for the departure trajectories, at initiation

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(2)Figure references apply to each of the five areas considered.

of burning for braking trajectories. The change in other trajectory parameters is shown in Figures 10 and 11.

The vehicle mass characteristics can be determined from Figures 12 and 13 in the section entitled "Brake to Mars Orbit."

SECTION V. CONCLUSIONS

From these parametric analyses, sufficient data are provided to enable the designer to make a preliminary design of a stage that will accomplish any one of the mission areas studied when that particular mission's requirements are defined.

SECTION VI. GRAPHIC PRESENTATION

DEPARTURE FROM EARTH ORBIT

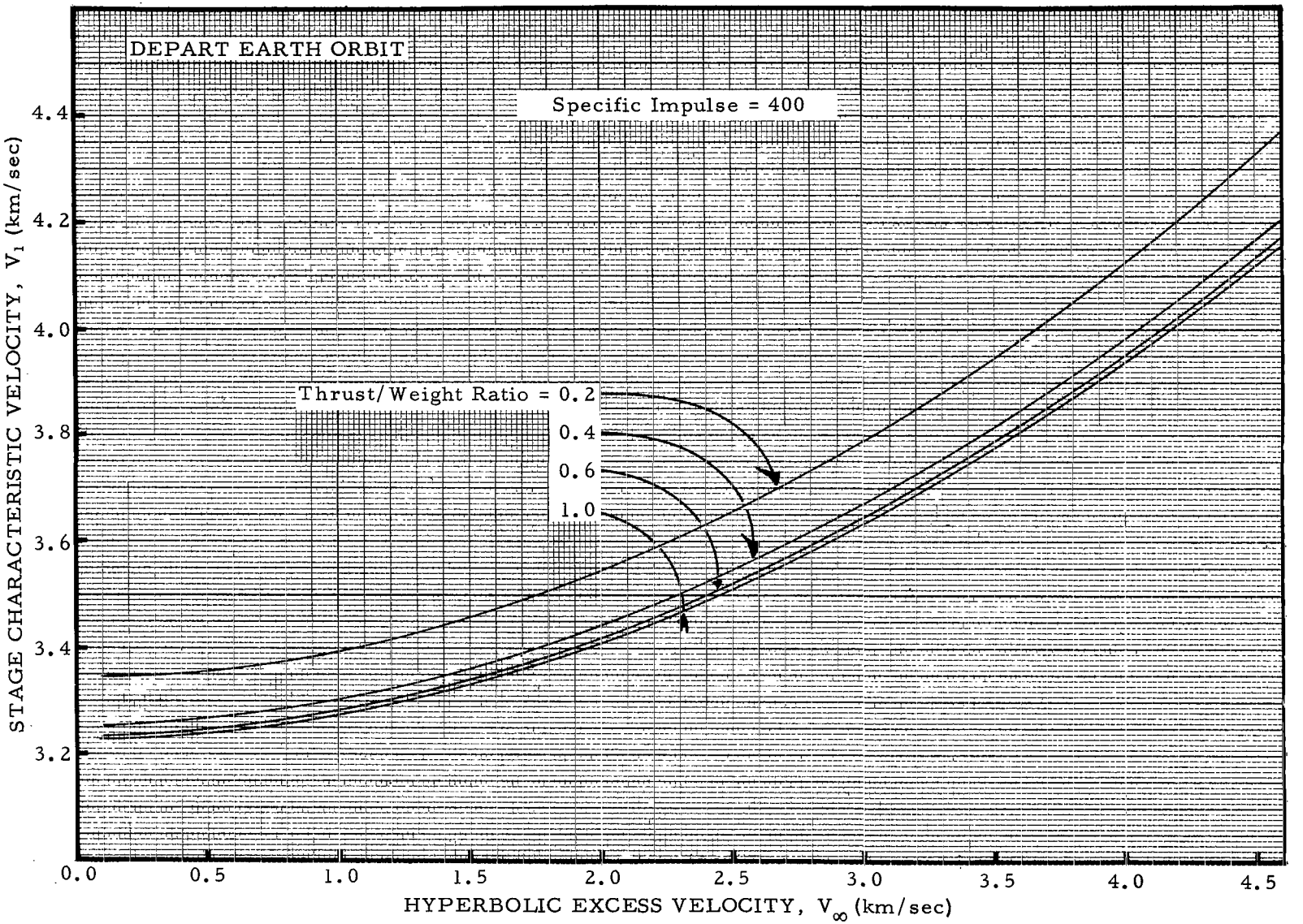


FIGURE 1a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

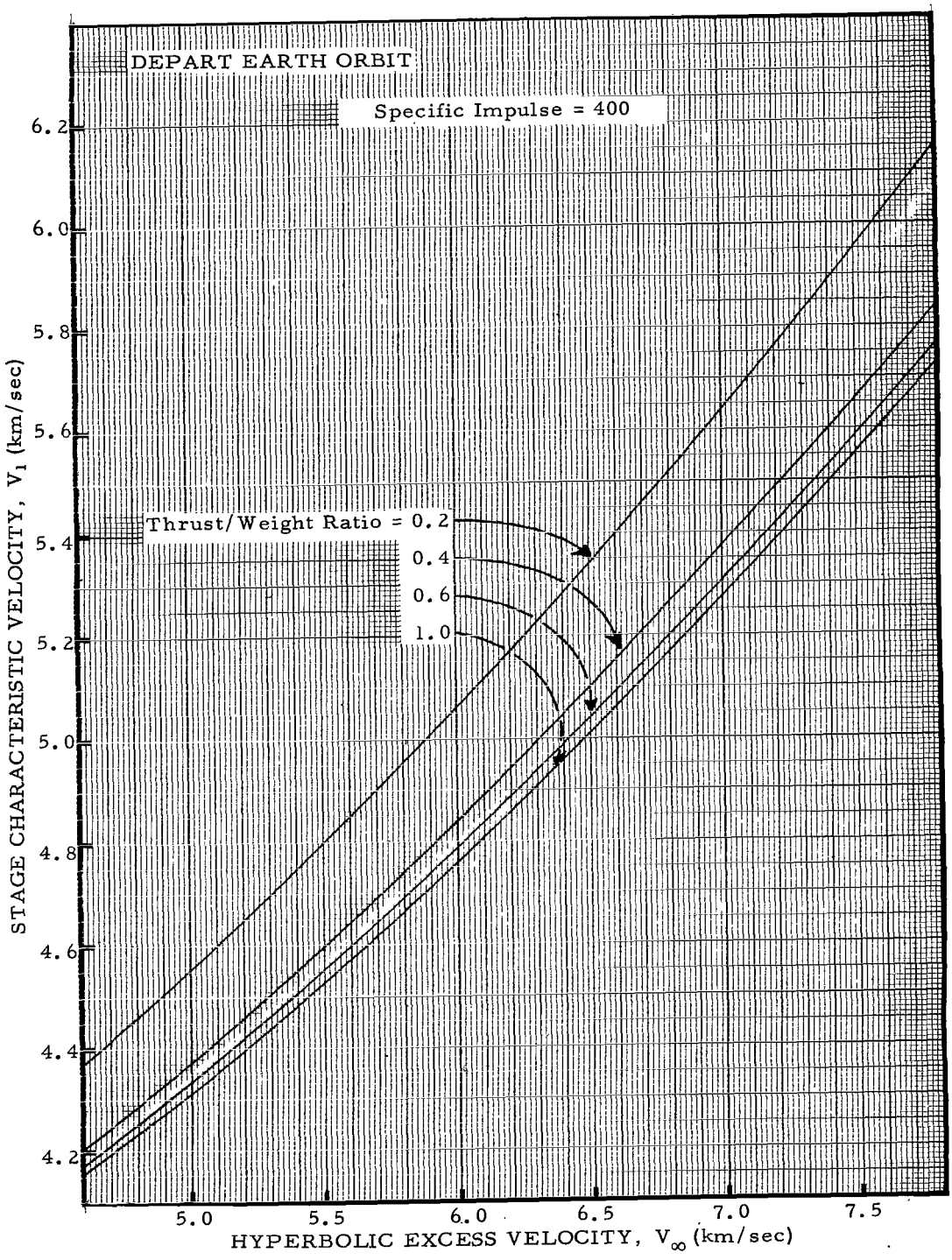


FIGURE 1b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

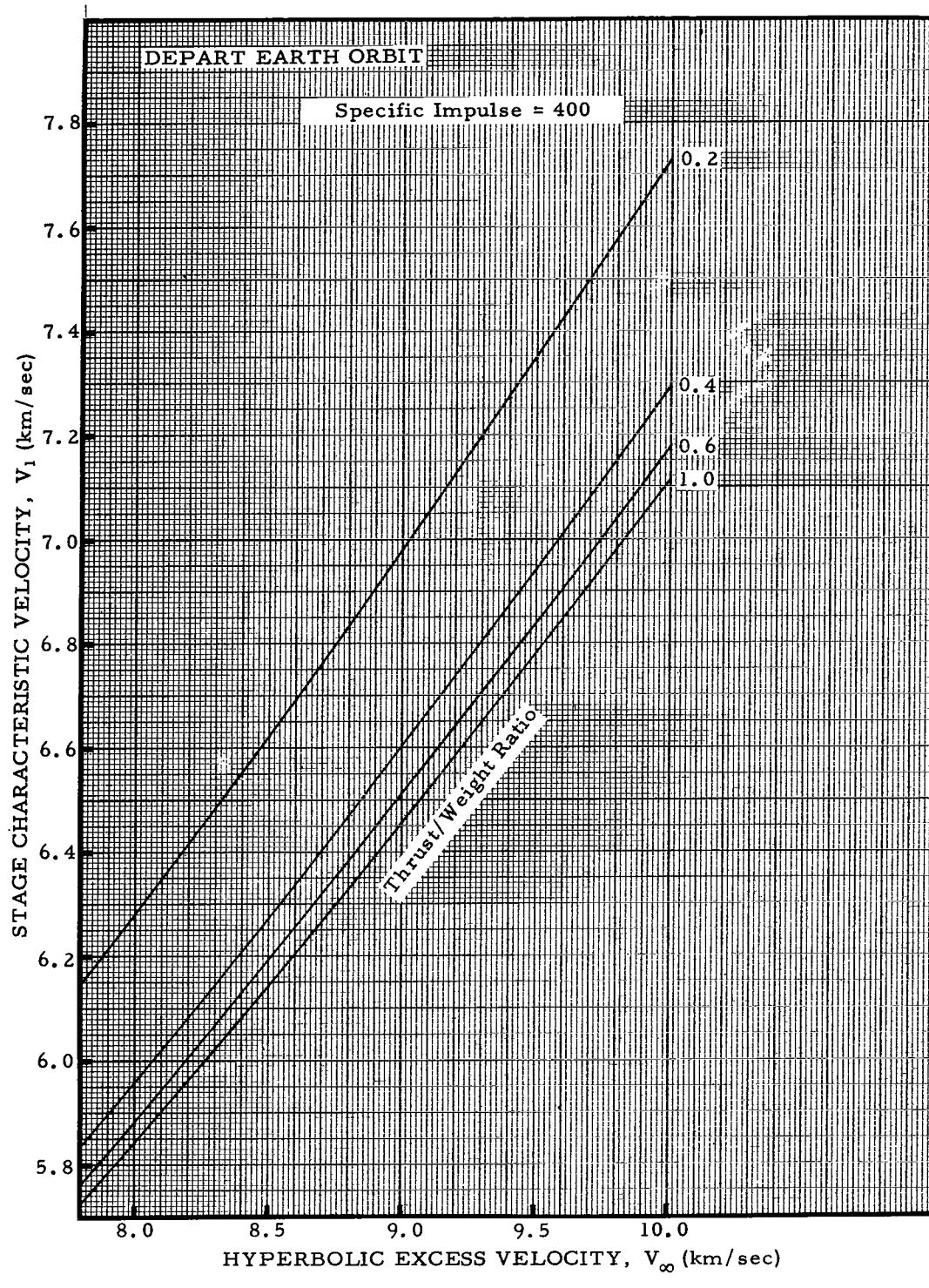


FIGURE 1c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

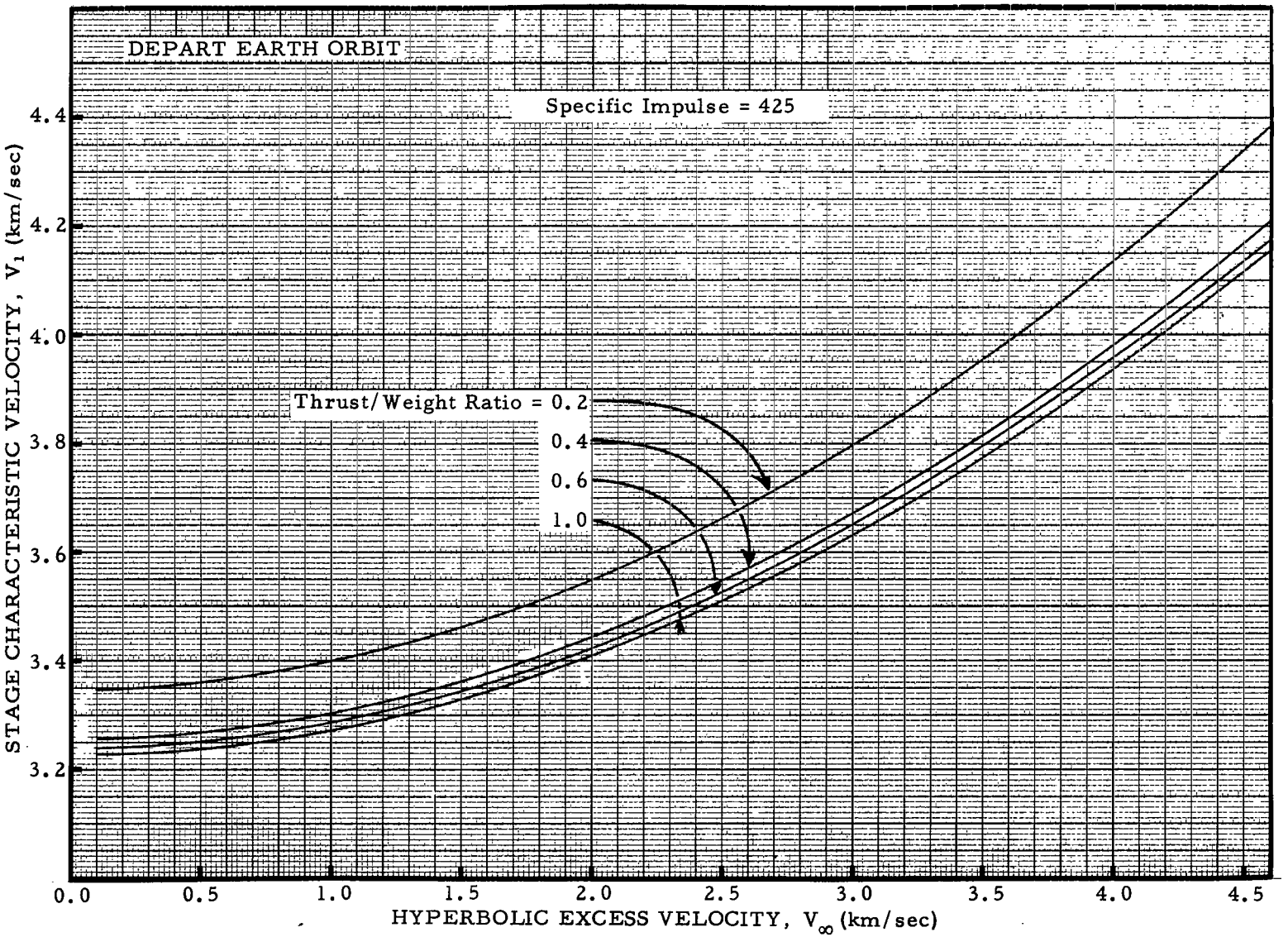


FIGURE 2a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

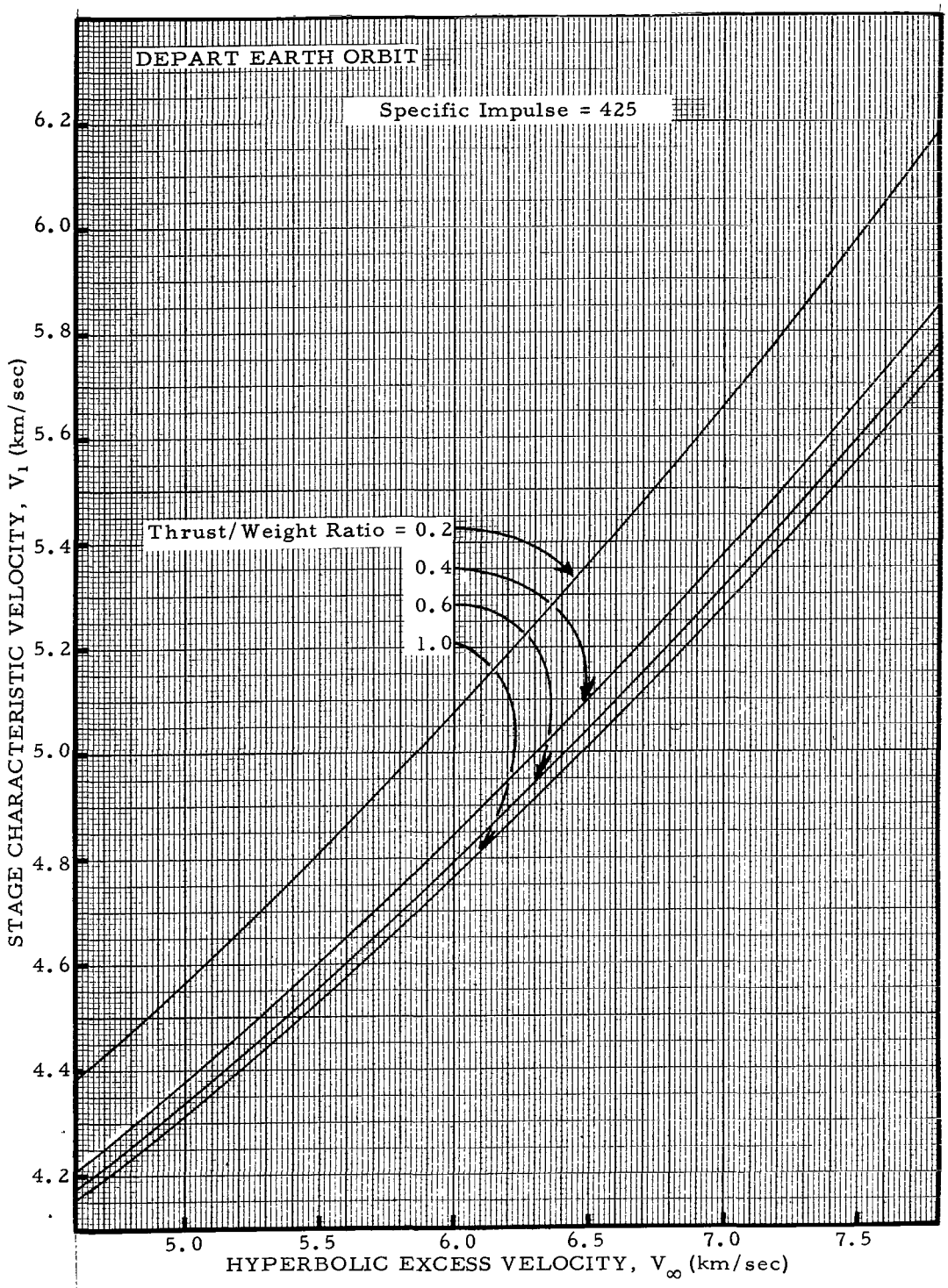


FIGURE 2b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

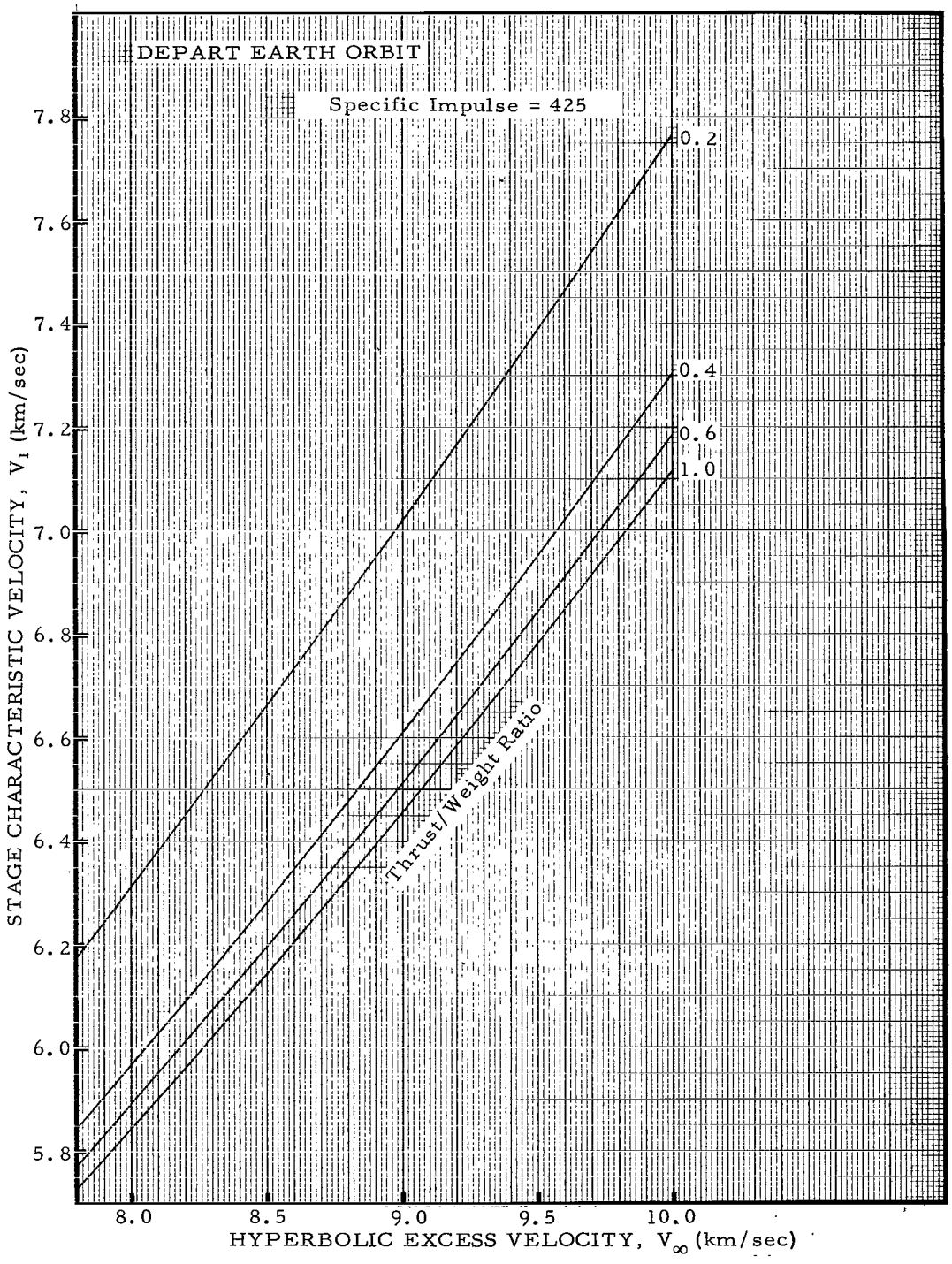


FIGURE 2c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

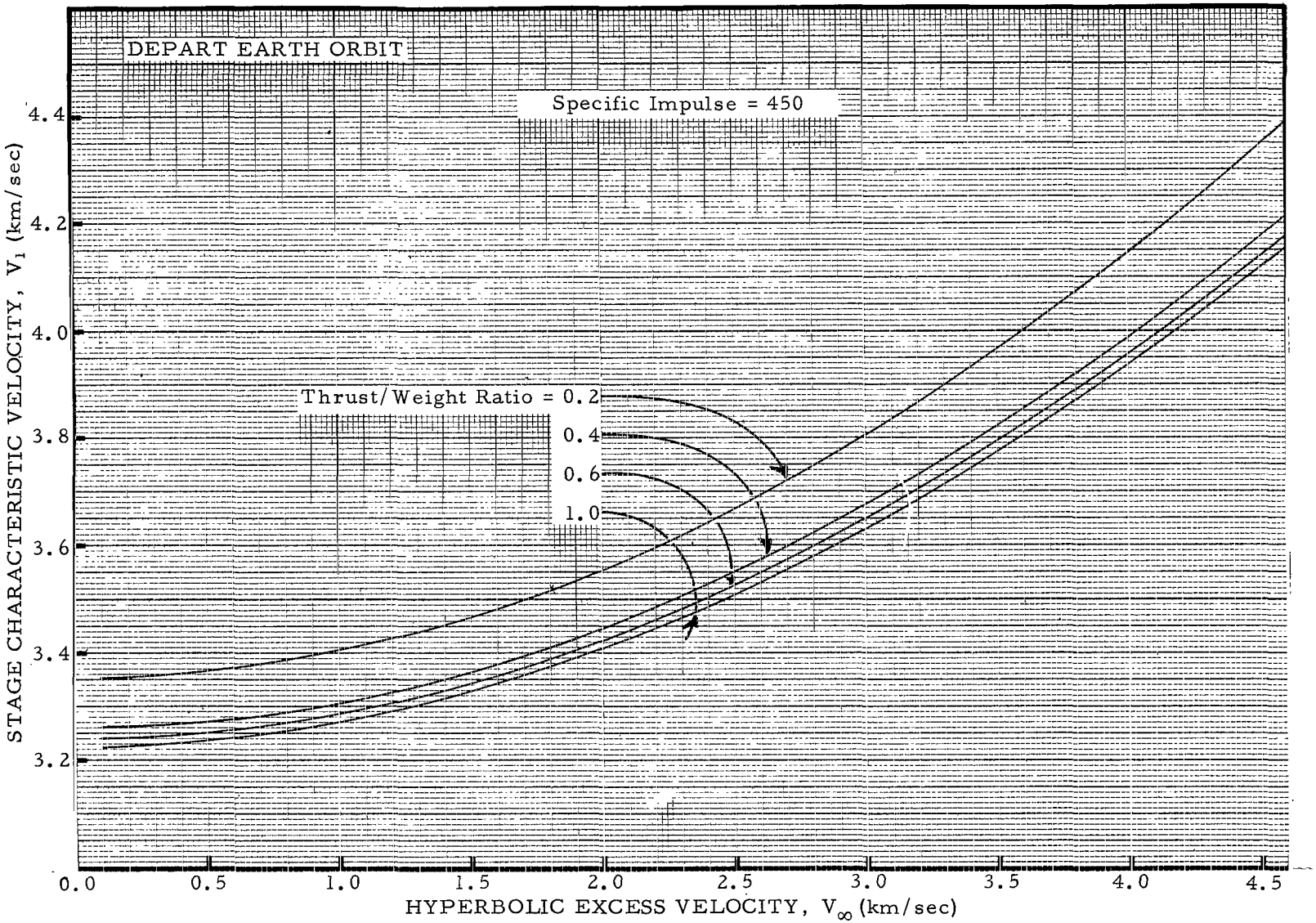


FIGURE 3a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

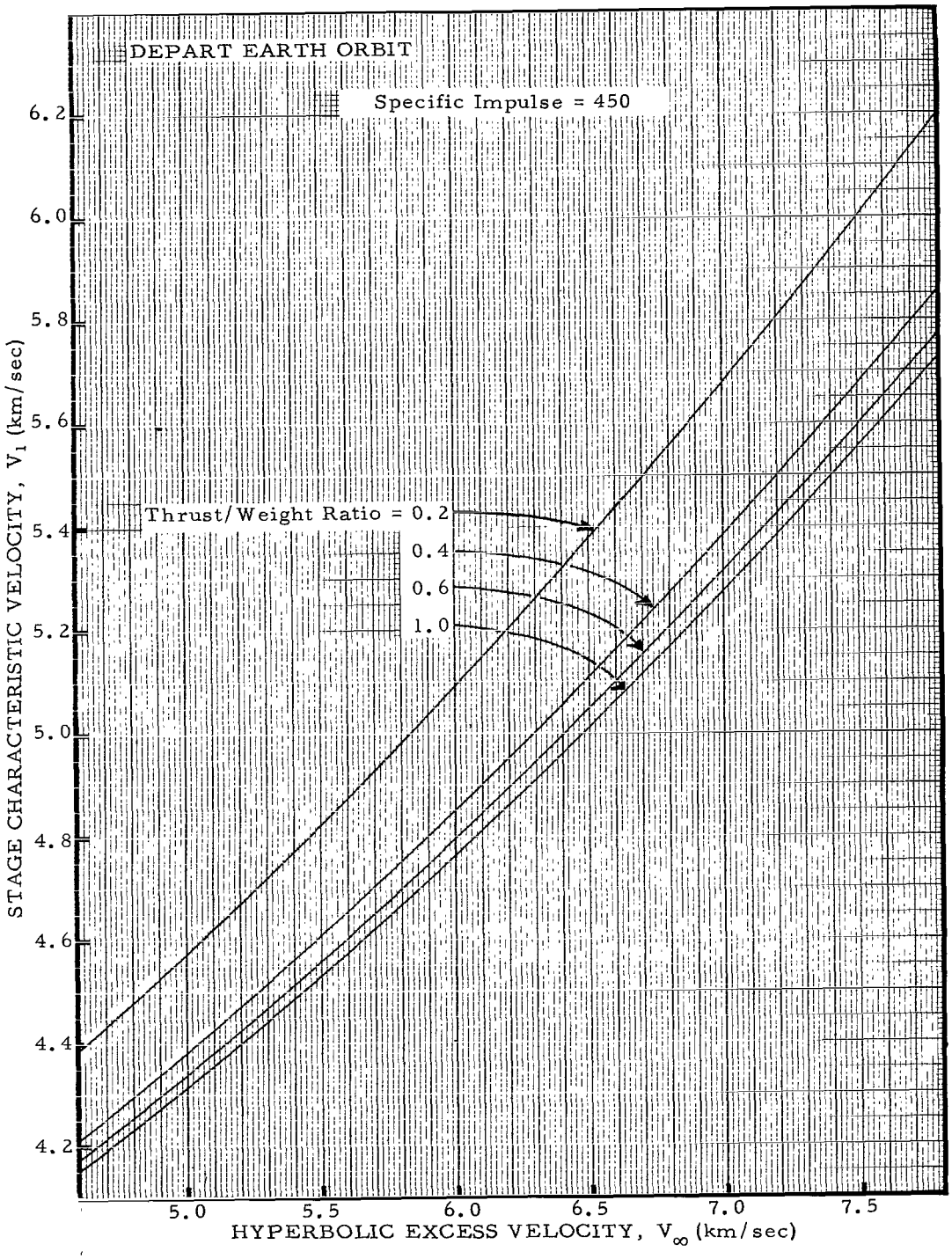


FIGURE 3b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

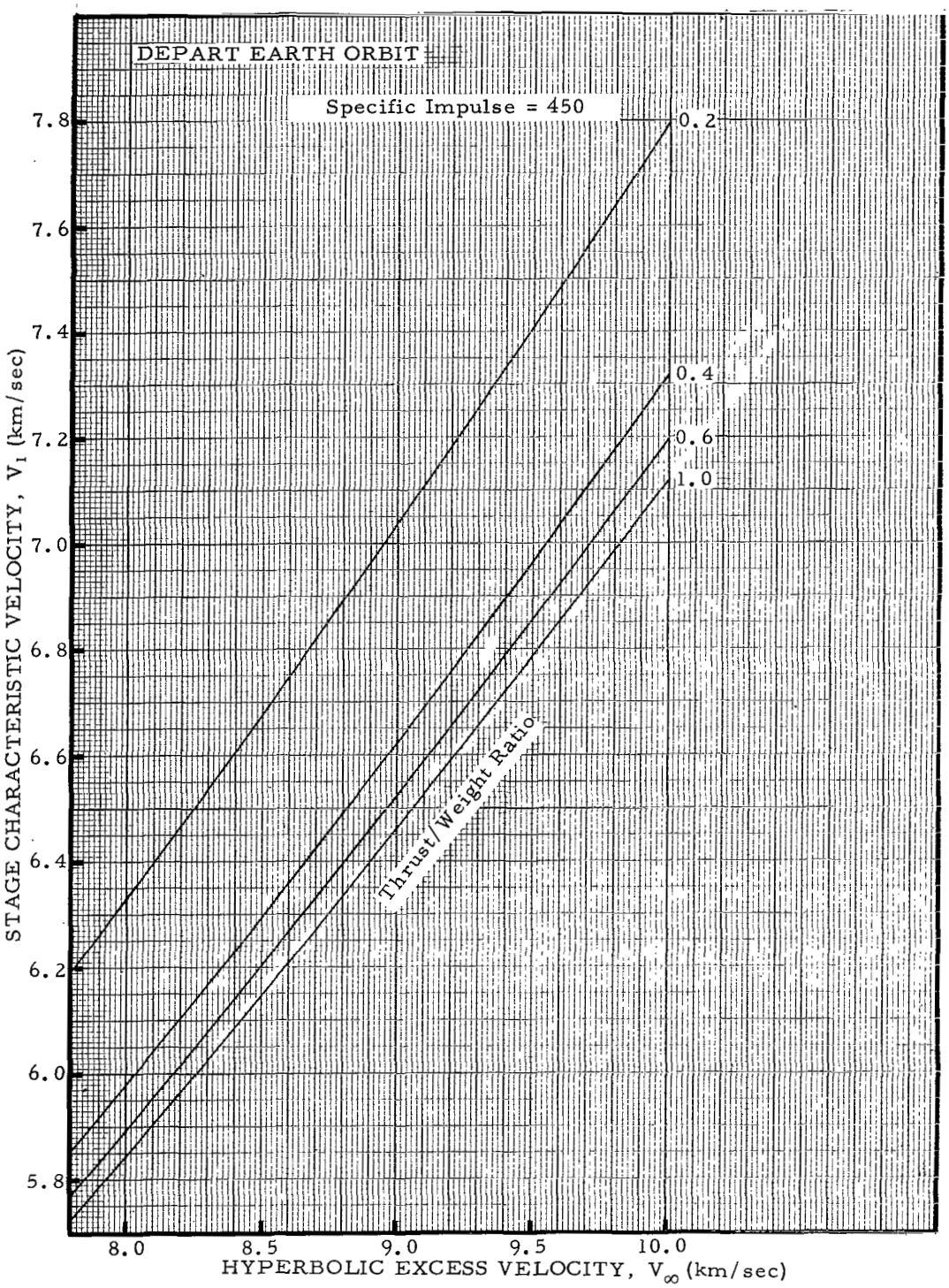


FIGURE 3c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

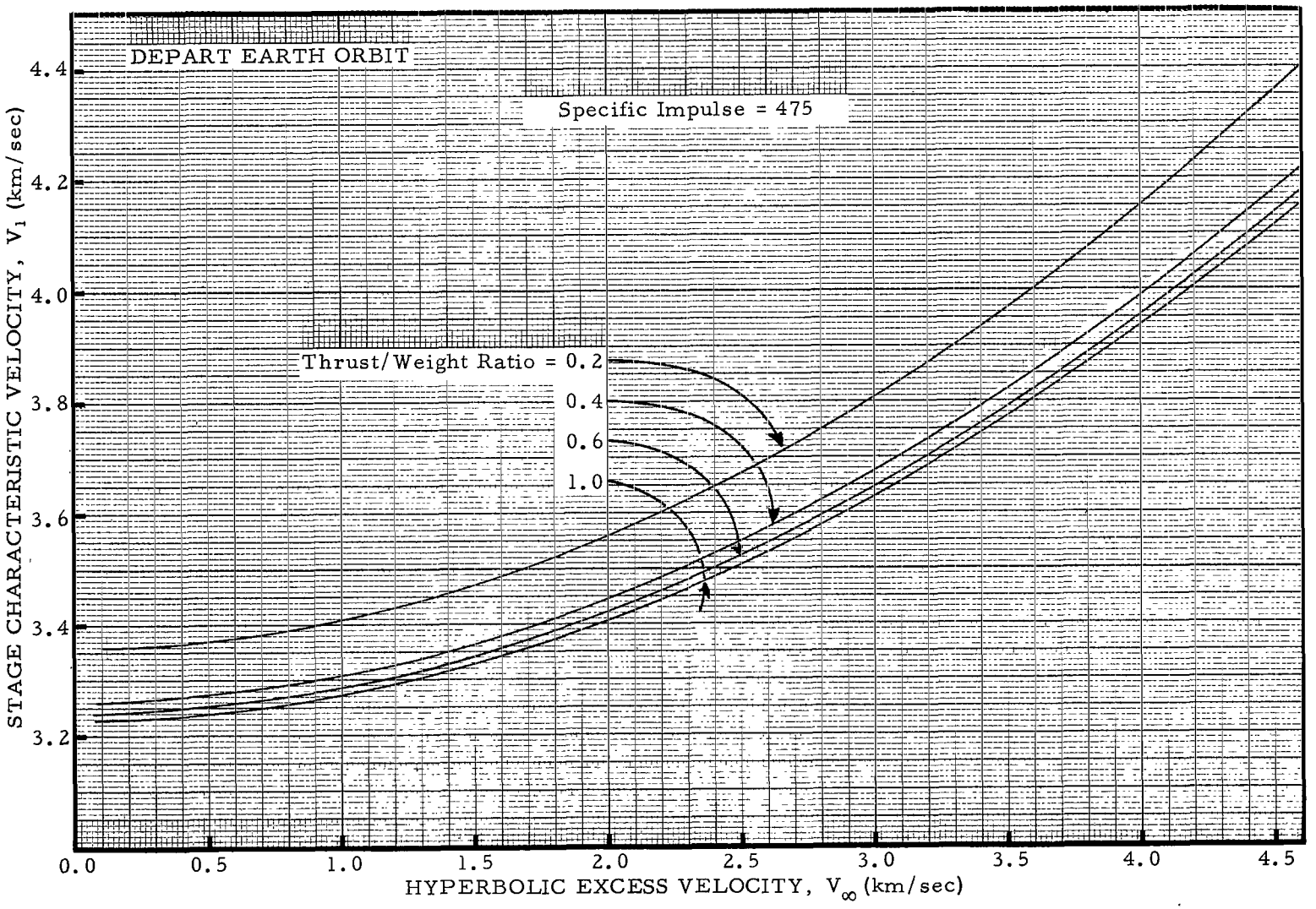


FIGURE 4a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

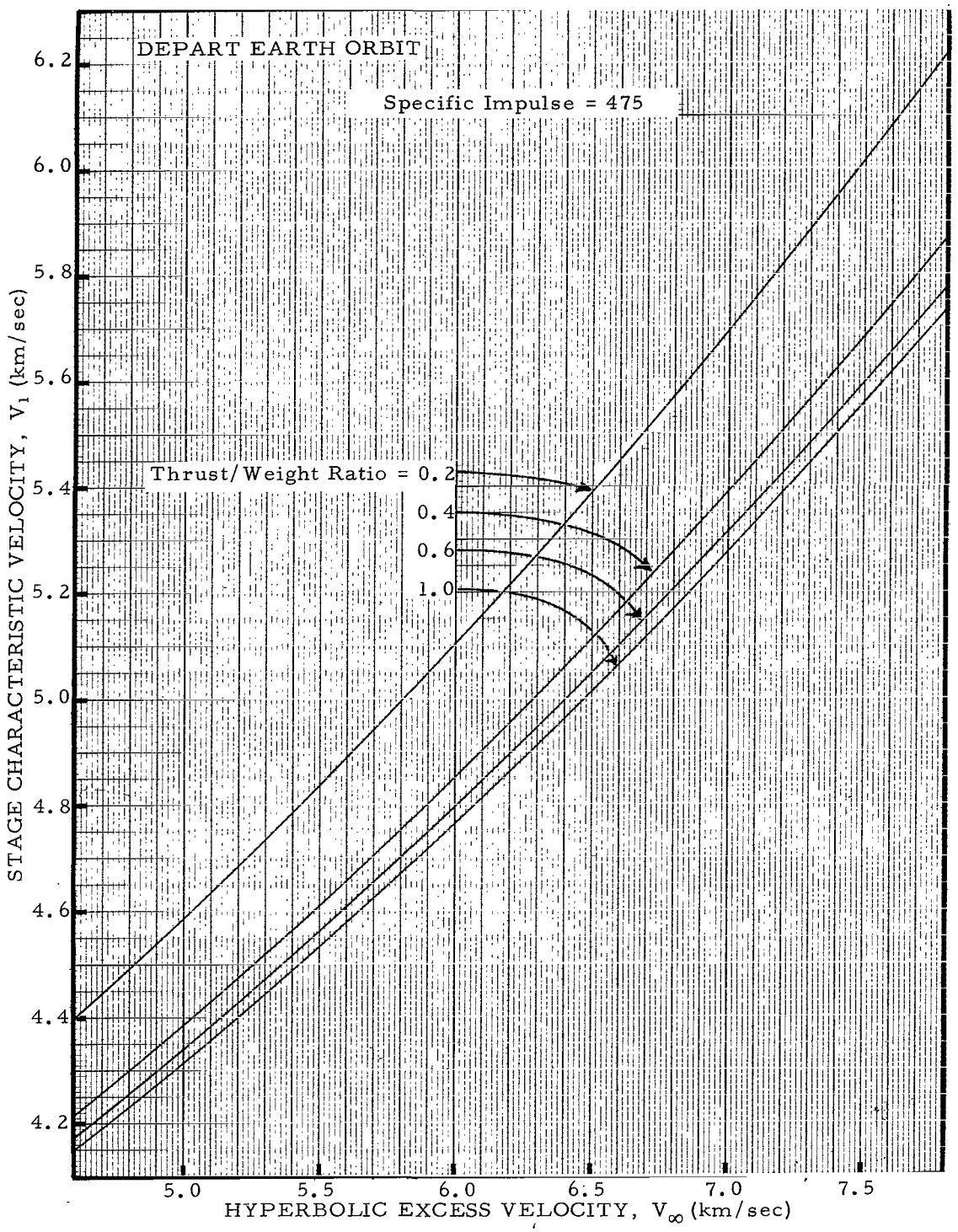


FIGURE 4b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

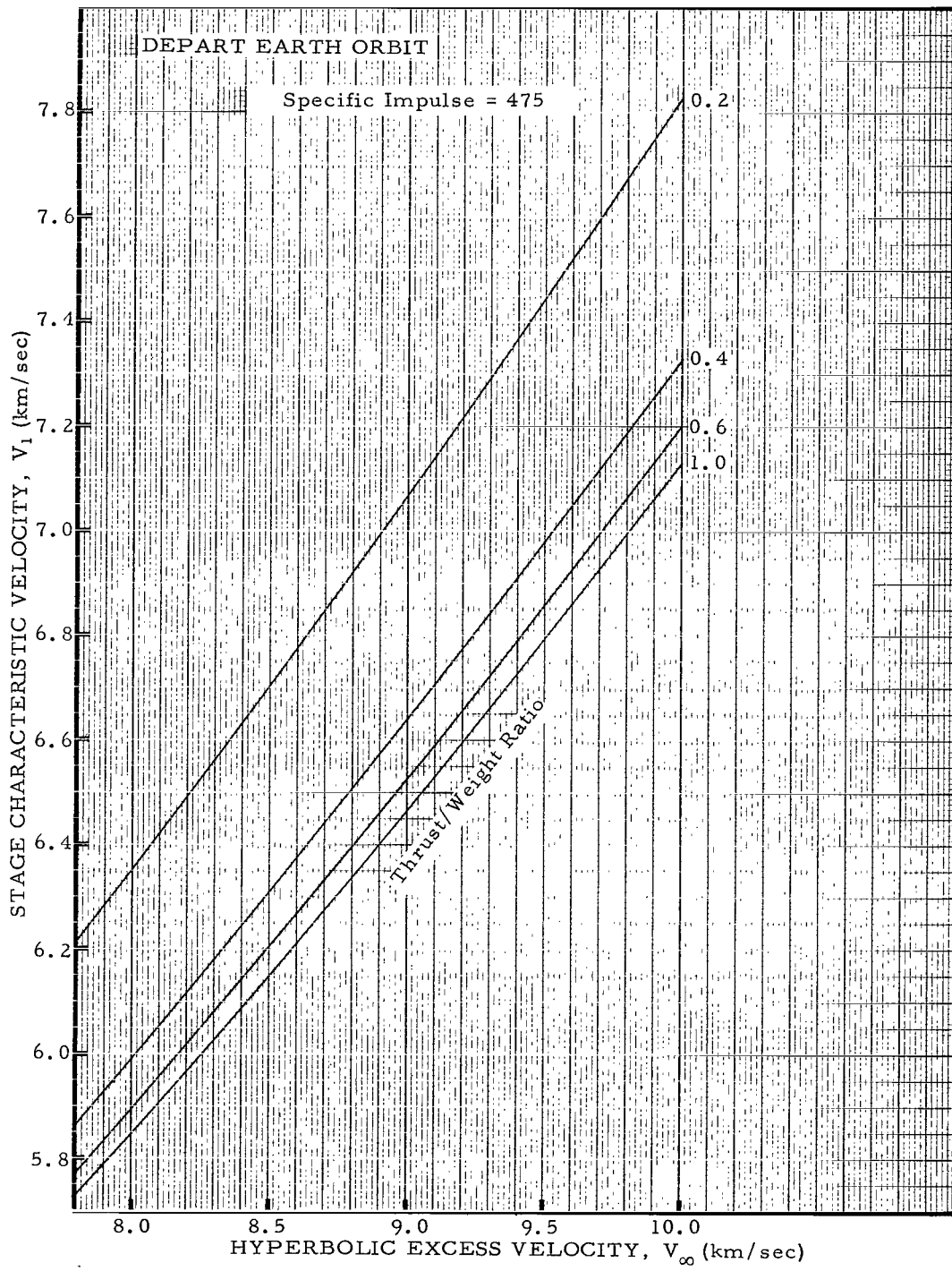


FIGURE 4c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

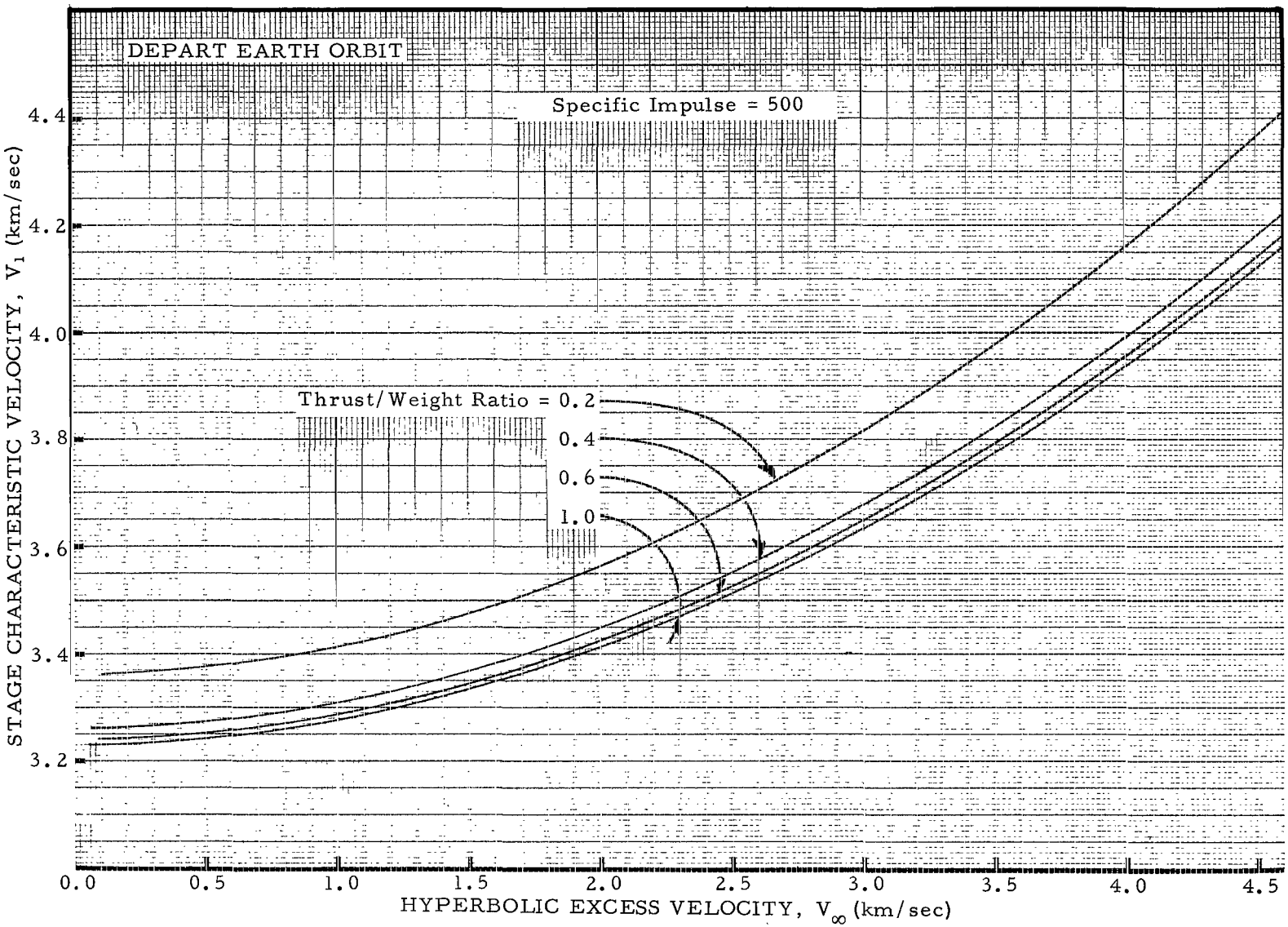


FIGURE 5a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

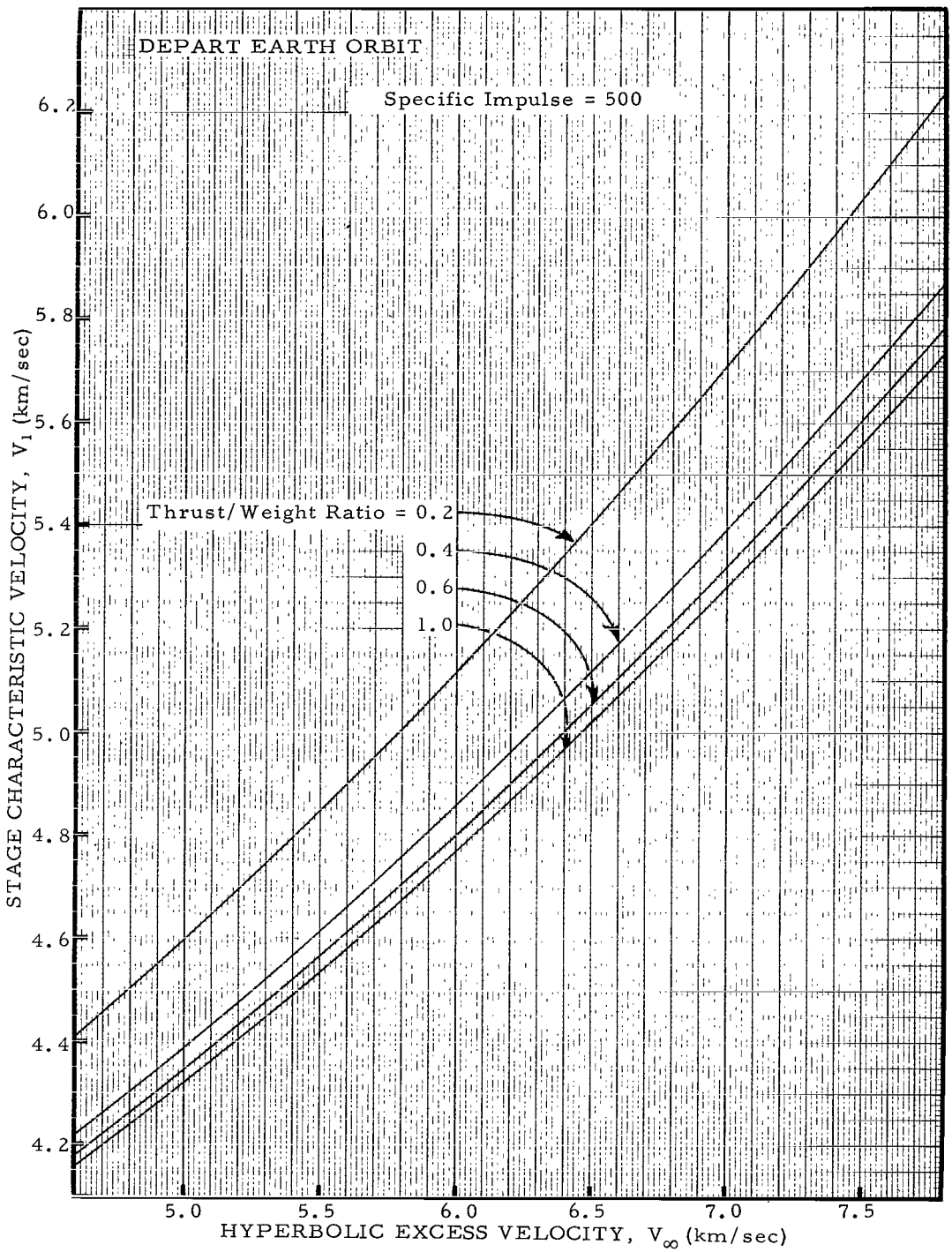


FIGURE 5b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

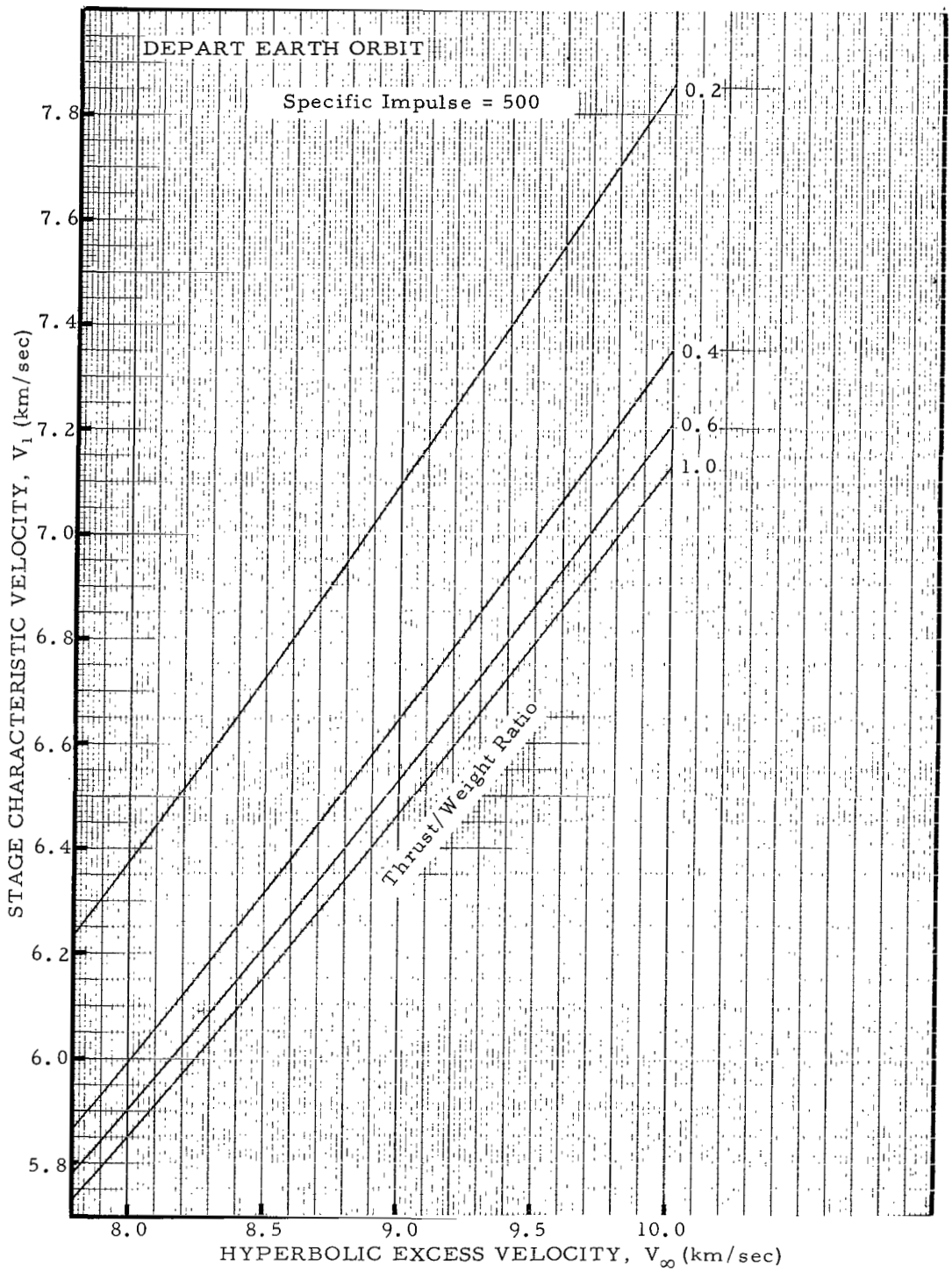


FIGURE 5c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

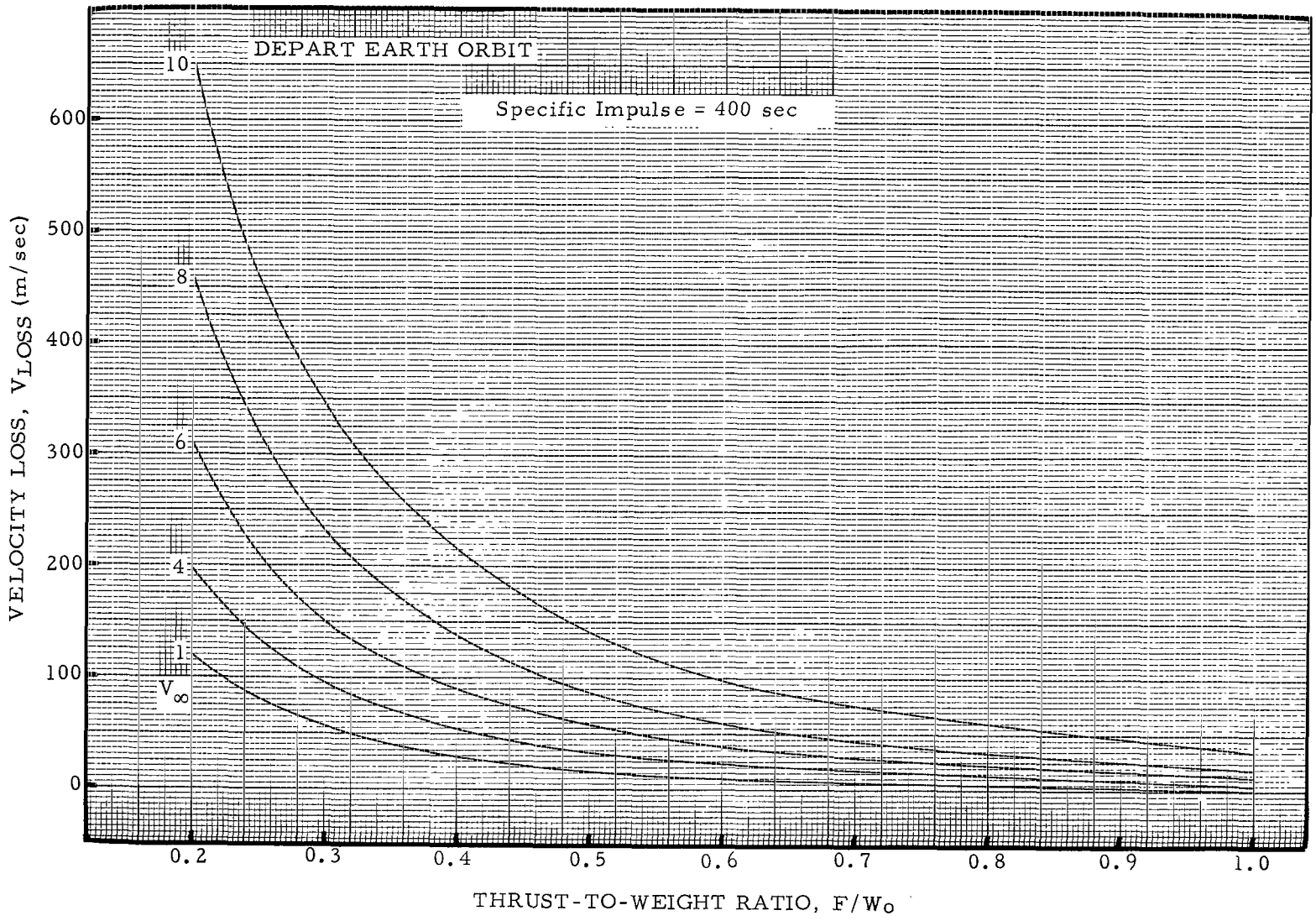


FIGURE 6. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

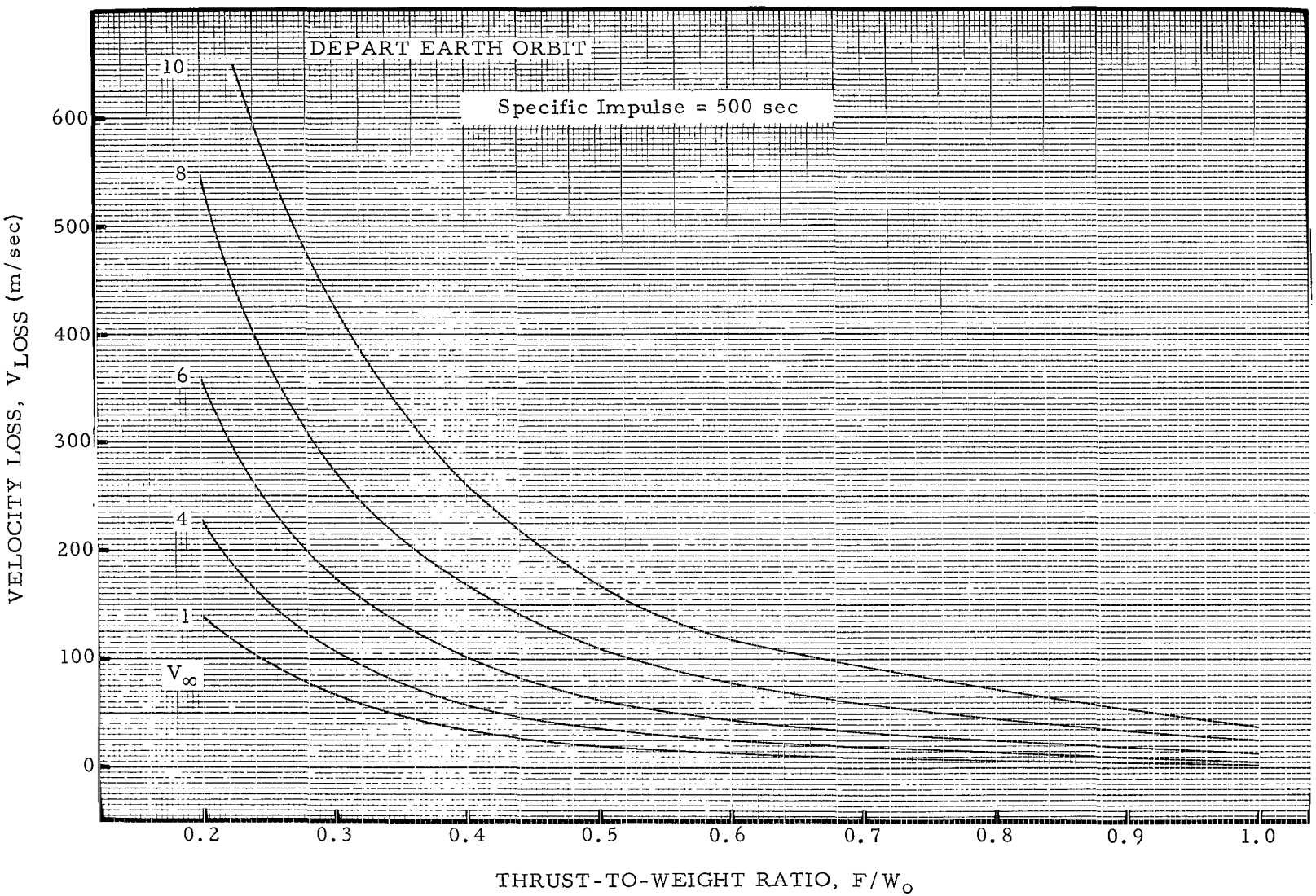


FIGURE 7. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

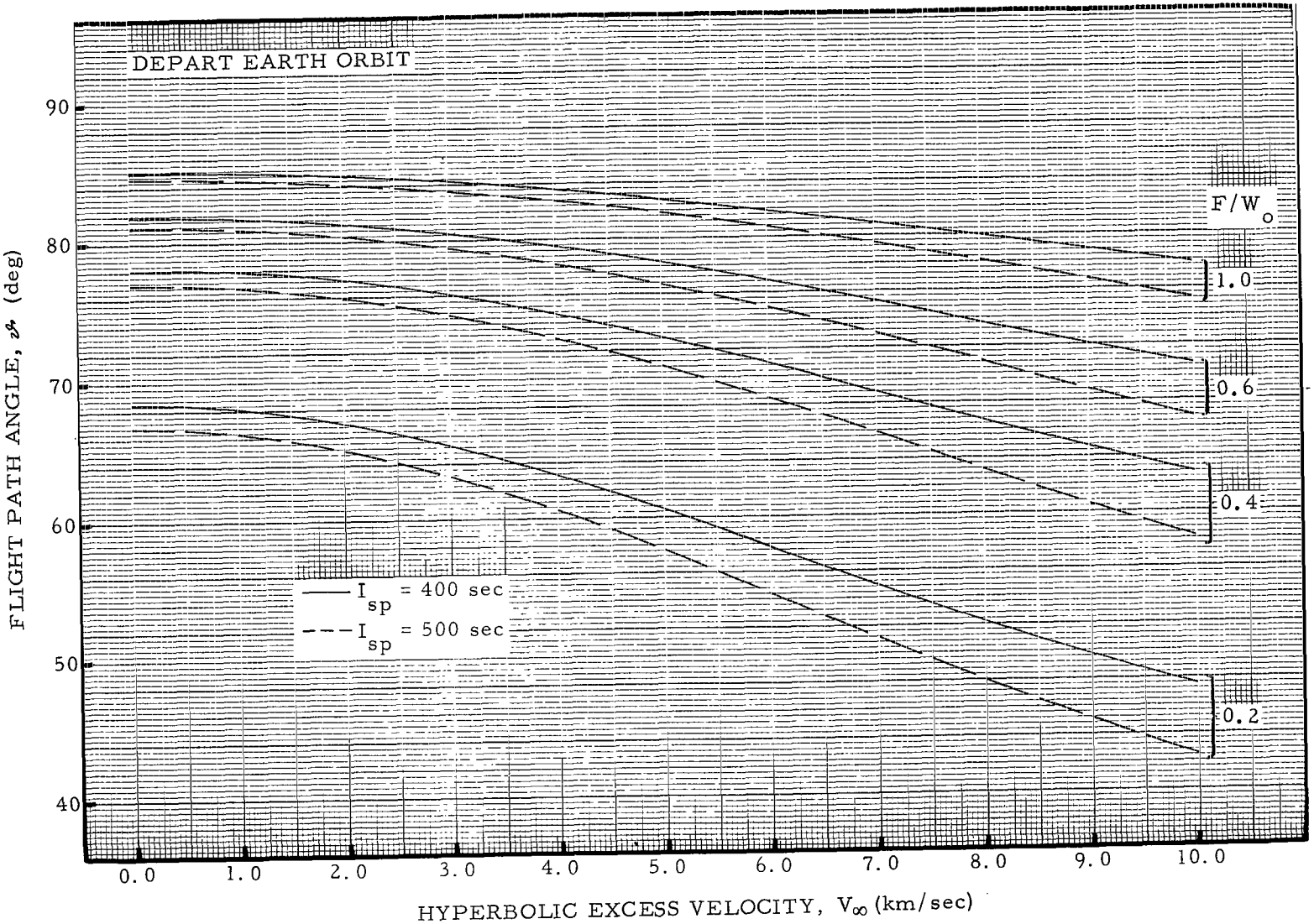


FIGURE 8. FLIGHT PATH ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

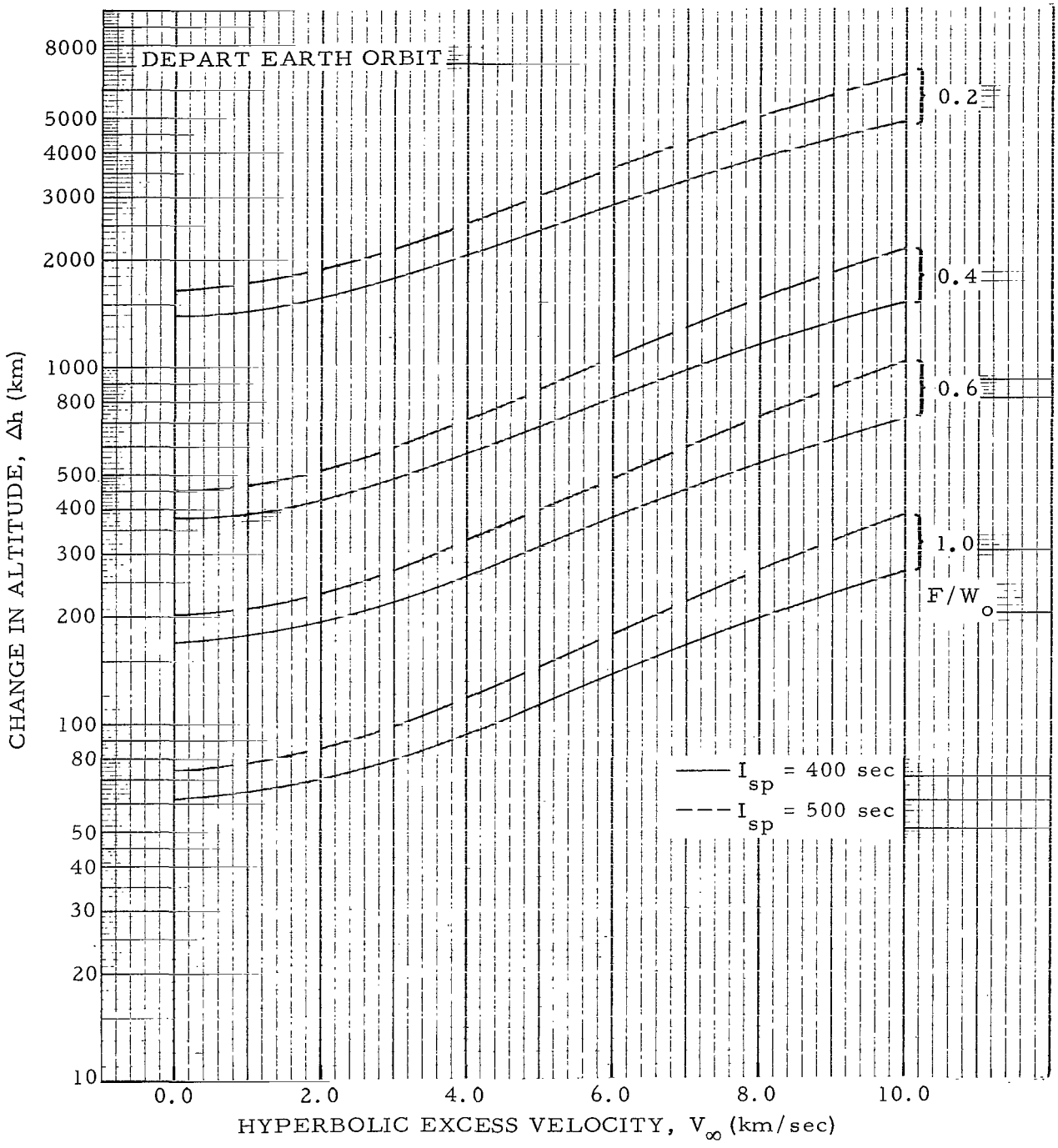


FIGURE 9. CHANGE IN ALTITUDE (km) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

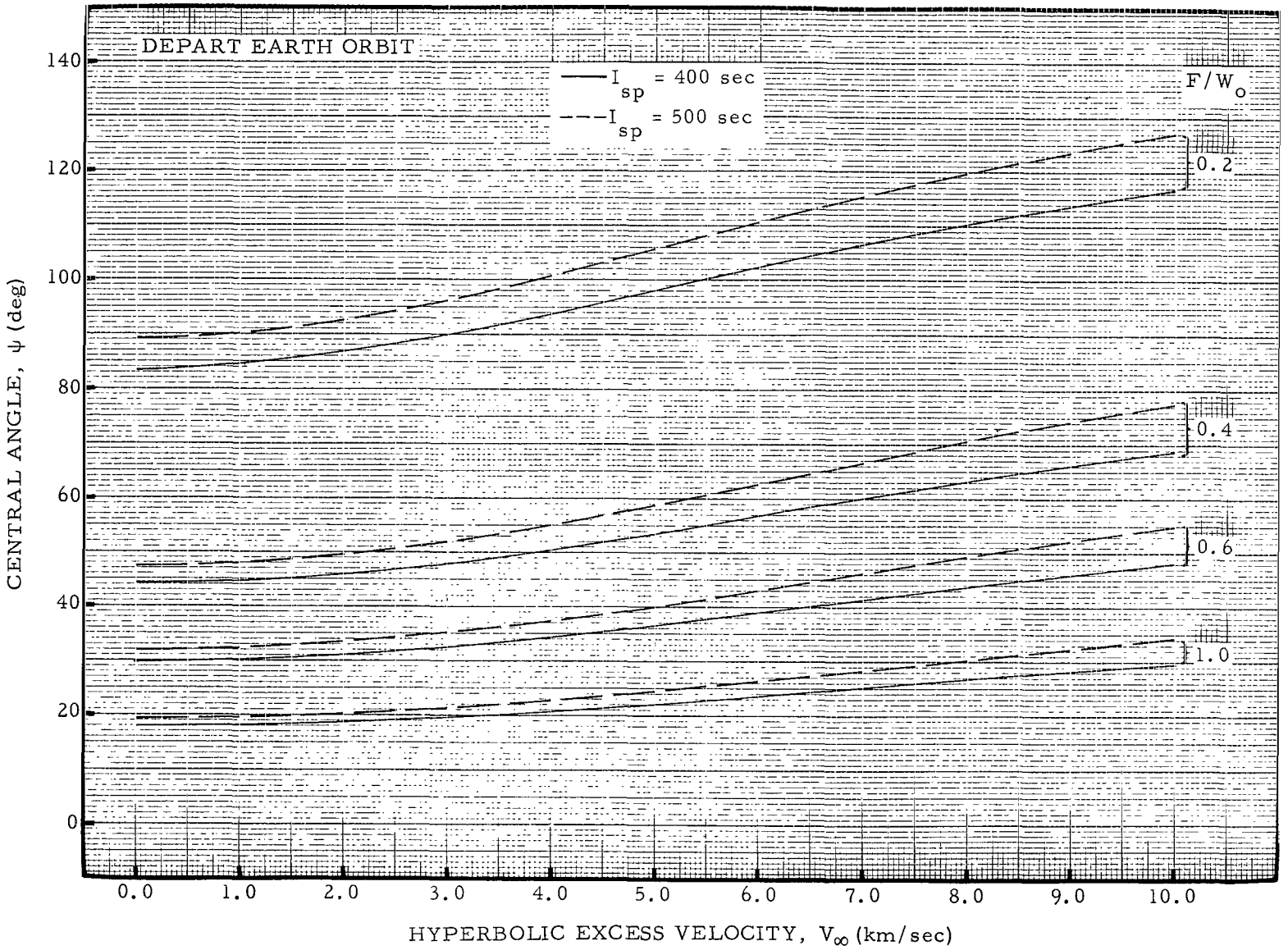


FIGURE 10. CENTRAL ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

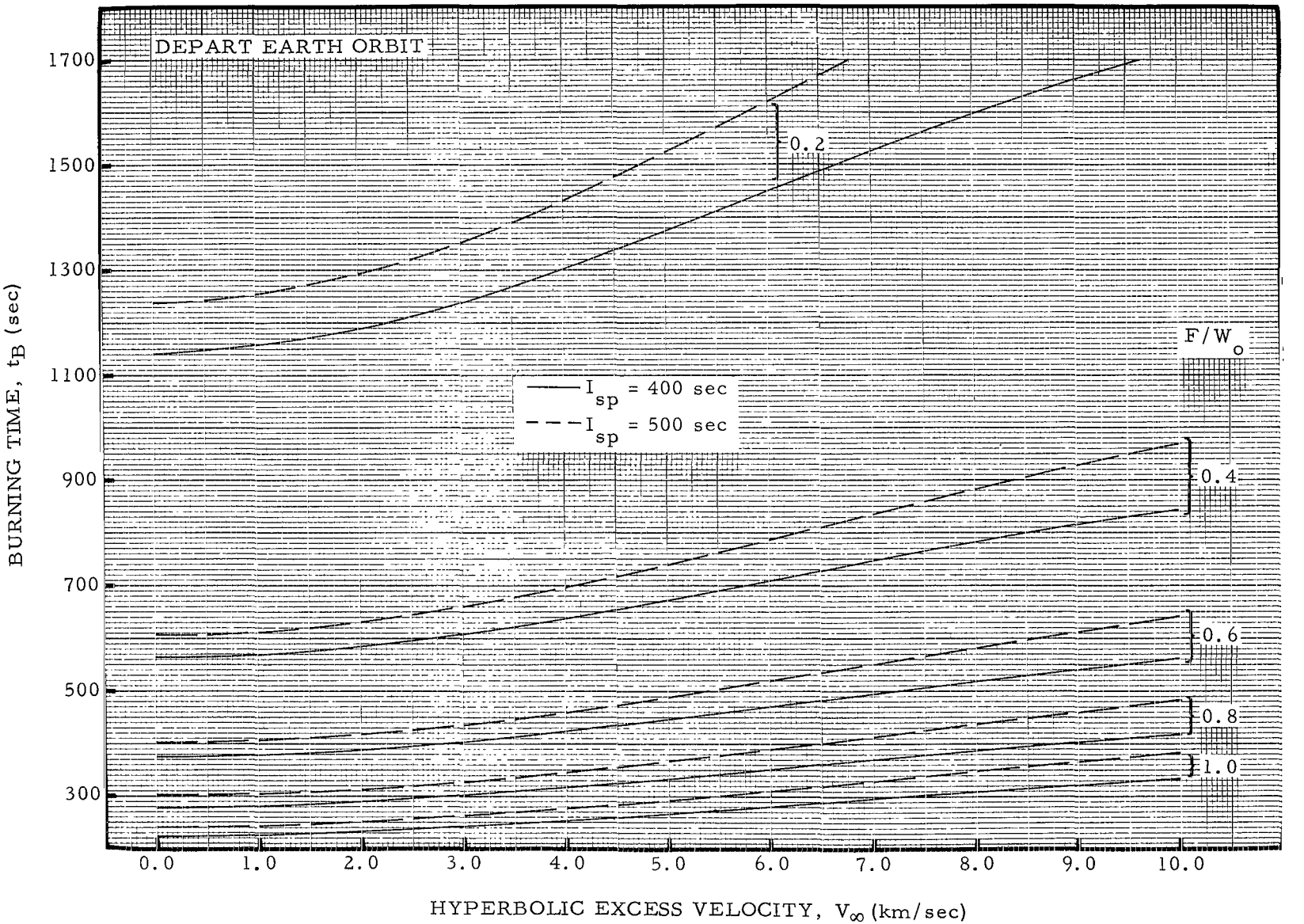


FIGURE 11. BURNING TIME (sec) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

DEPARTURE FROM VENUS ORBIT

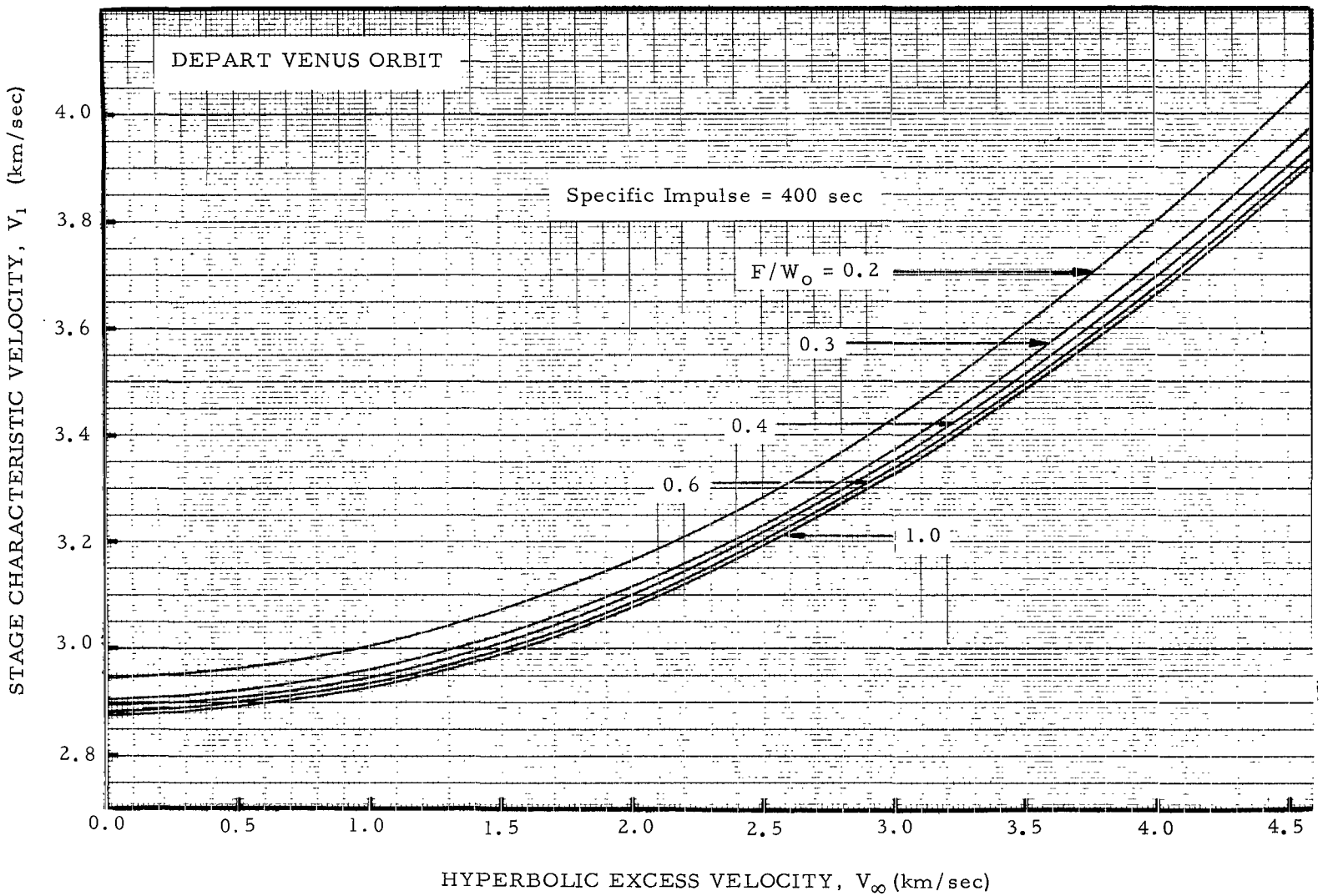


FIGURE 1a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

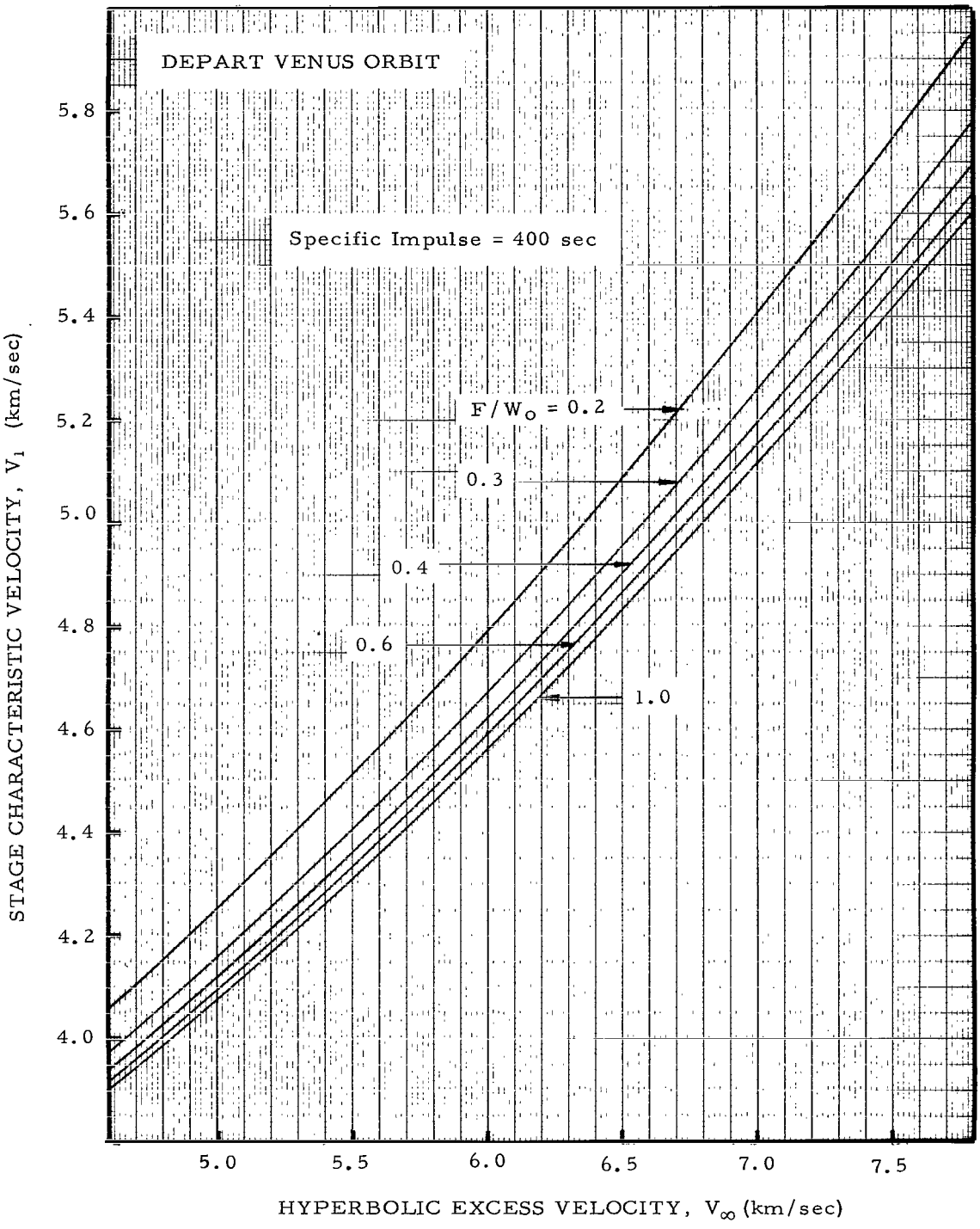


FIGURE 1b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

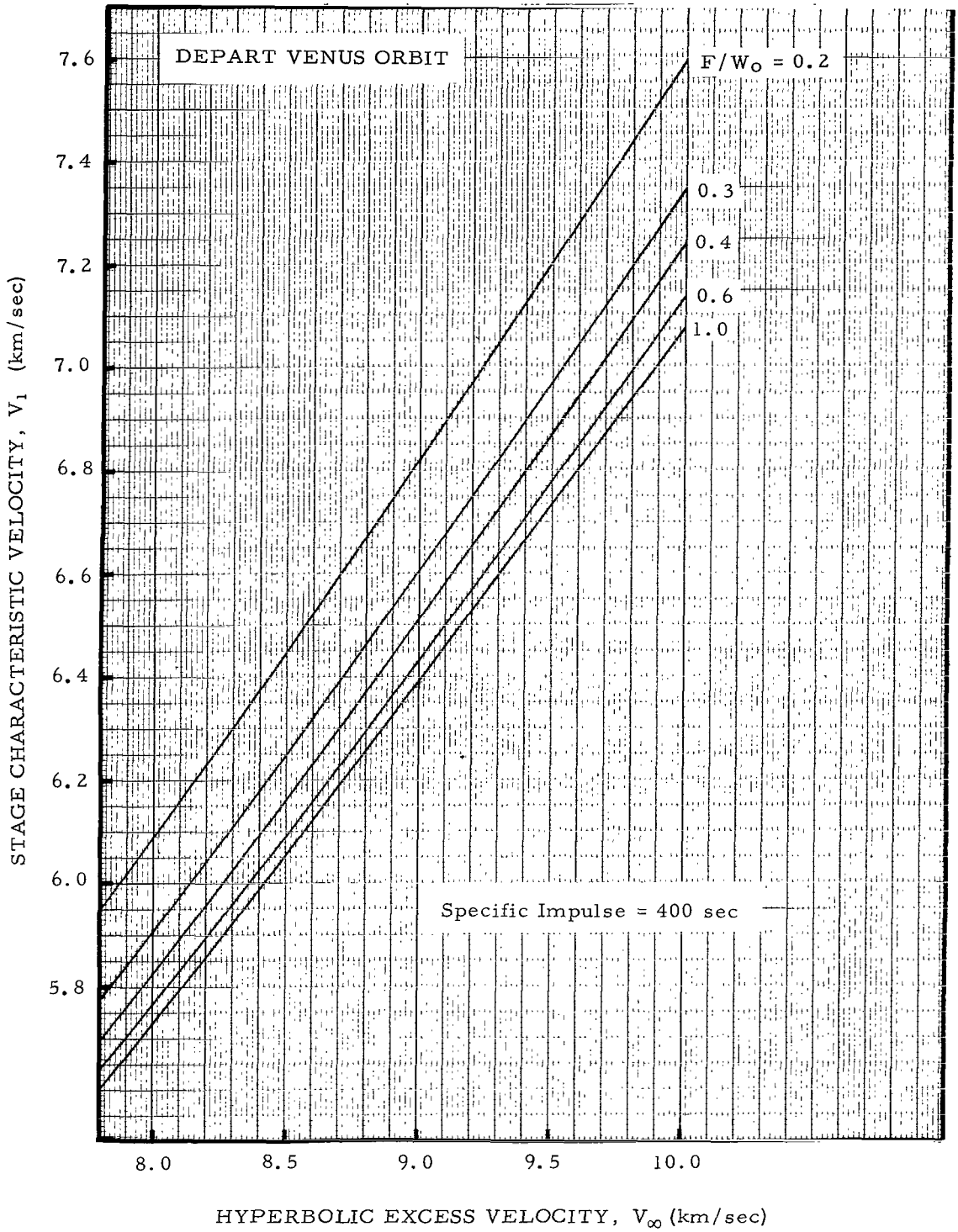


FIGURE 1c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

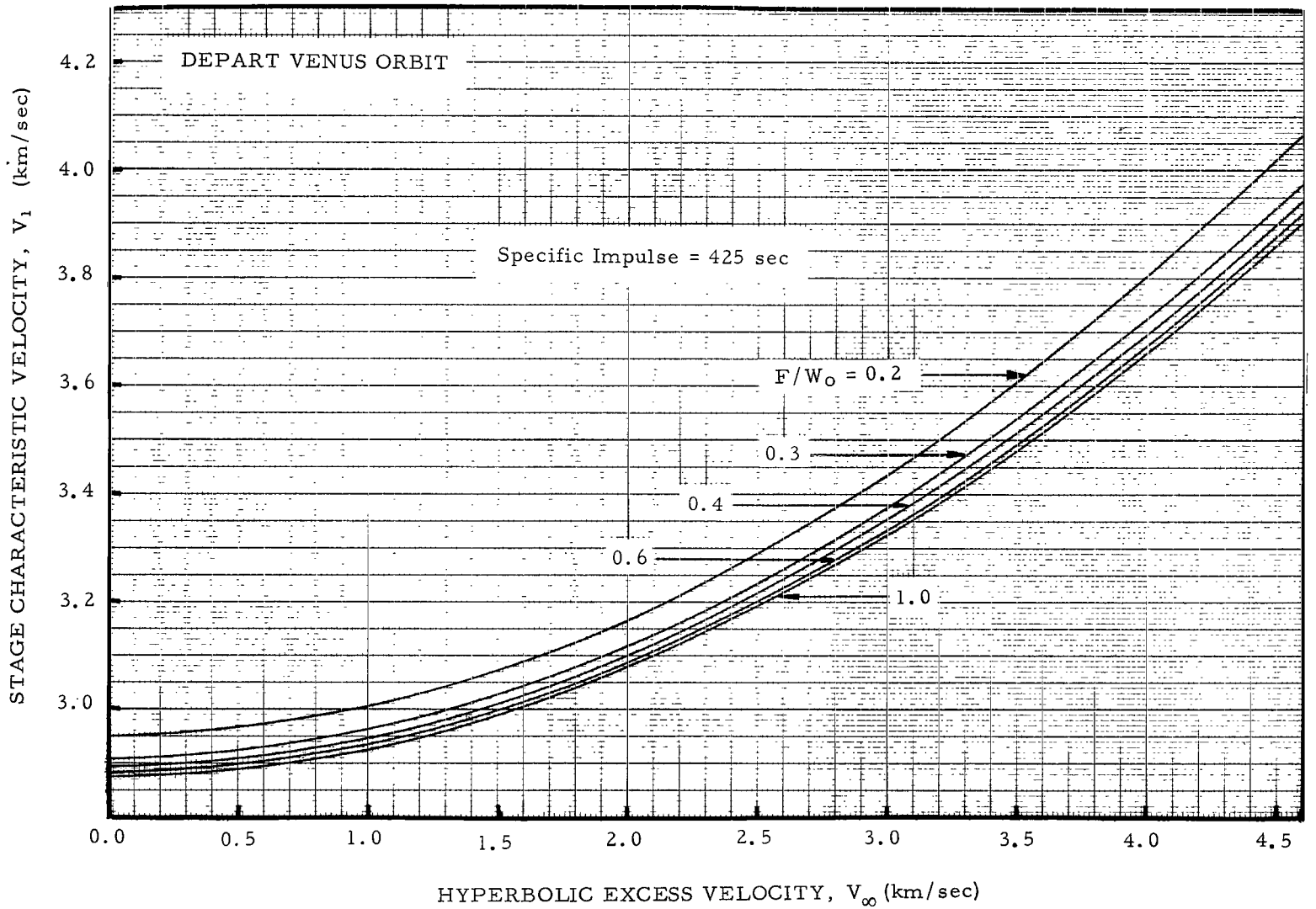
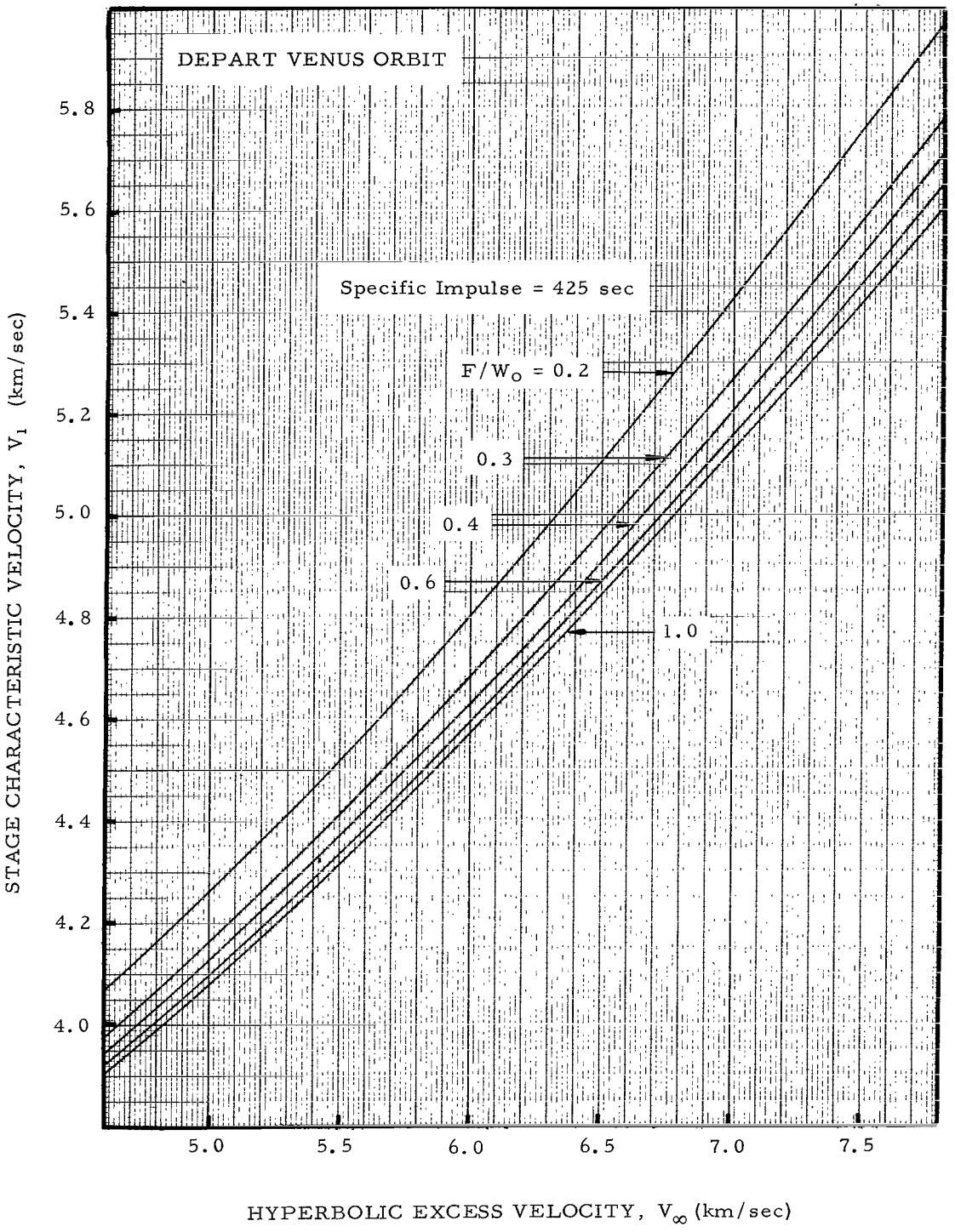


FIGURE 2a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS



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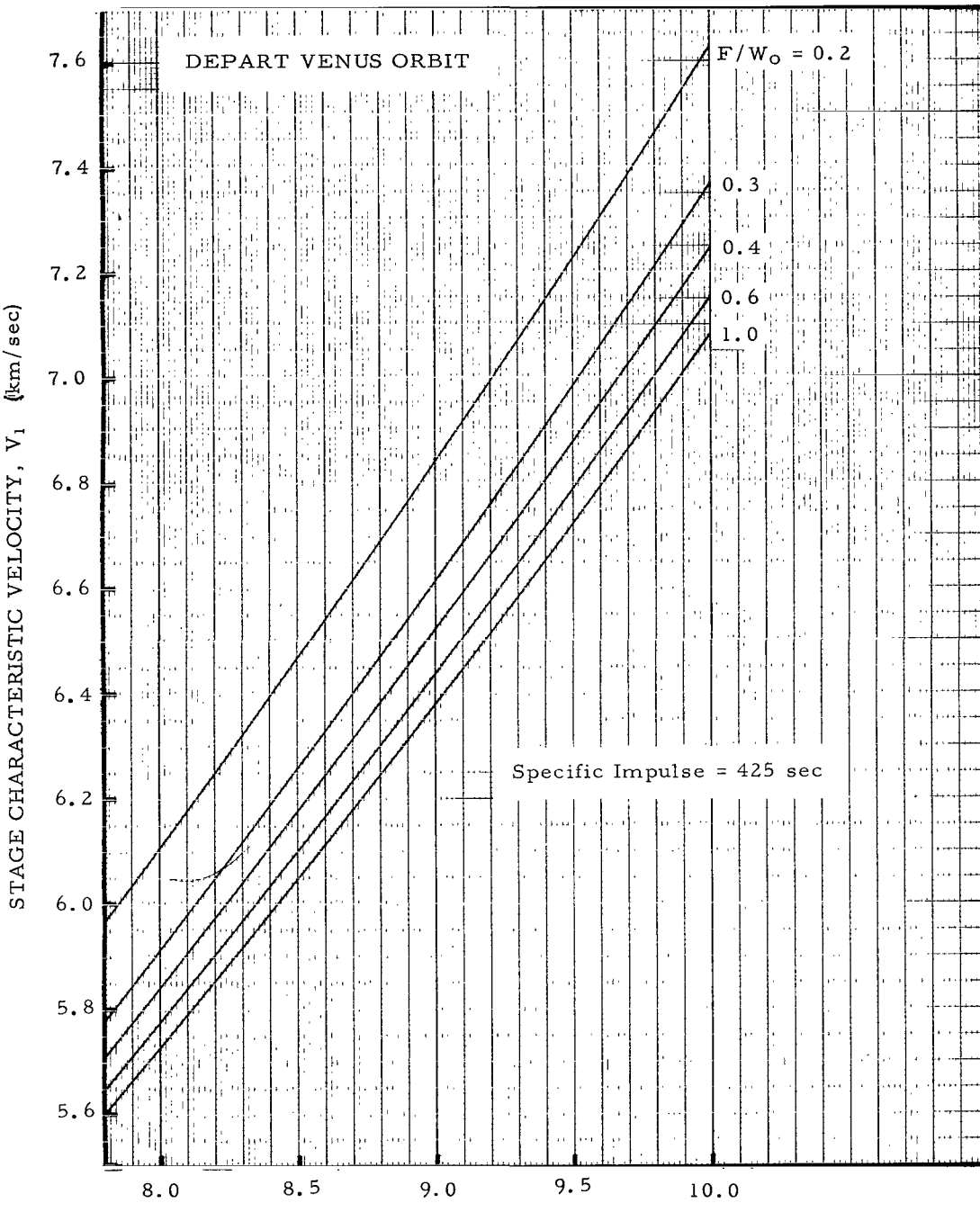

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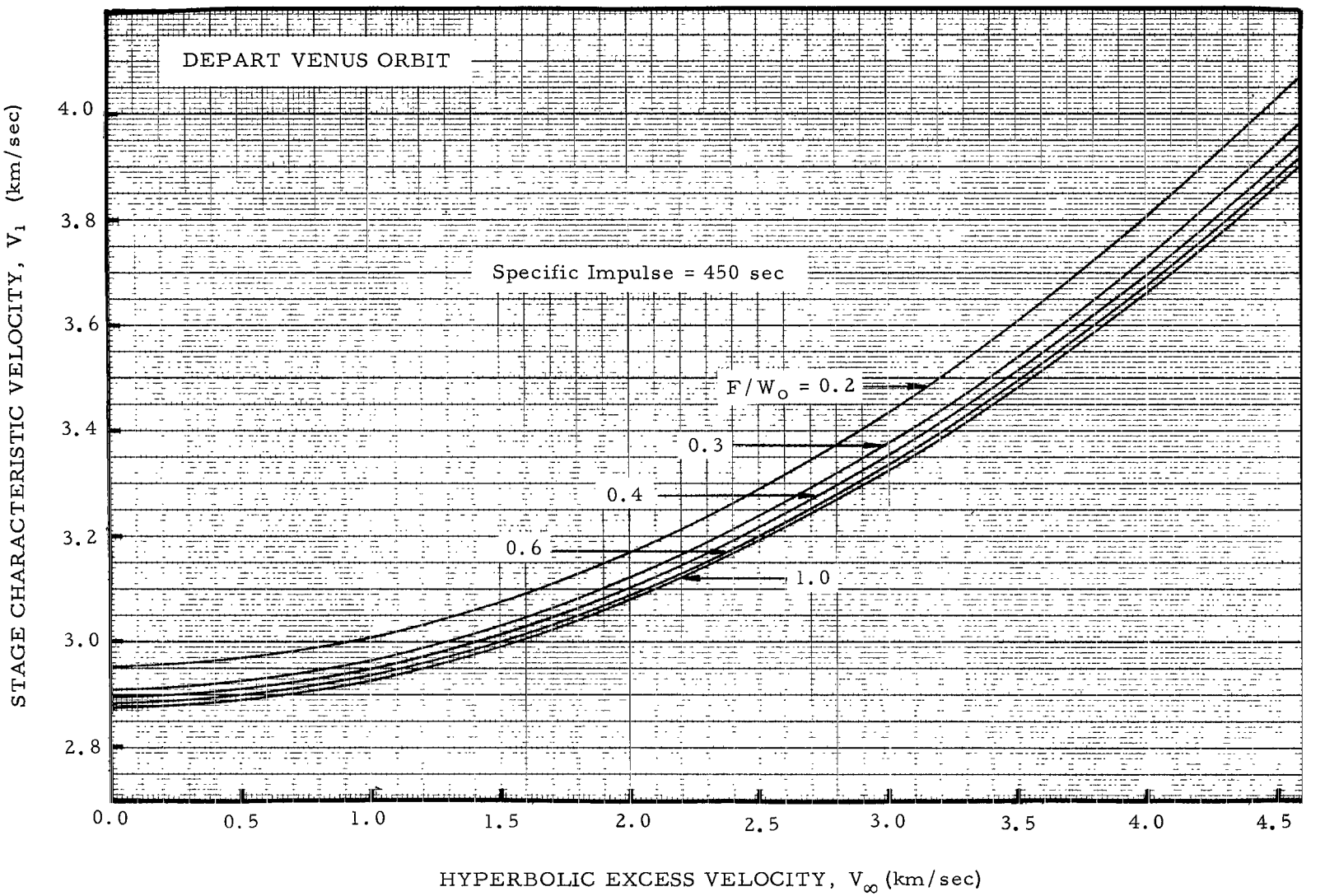

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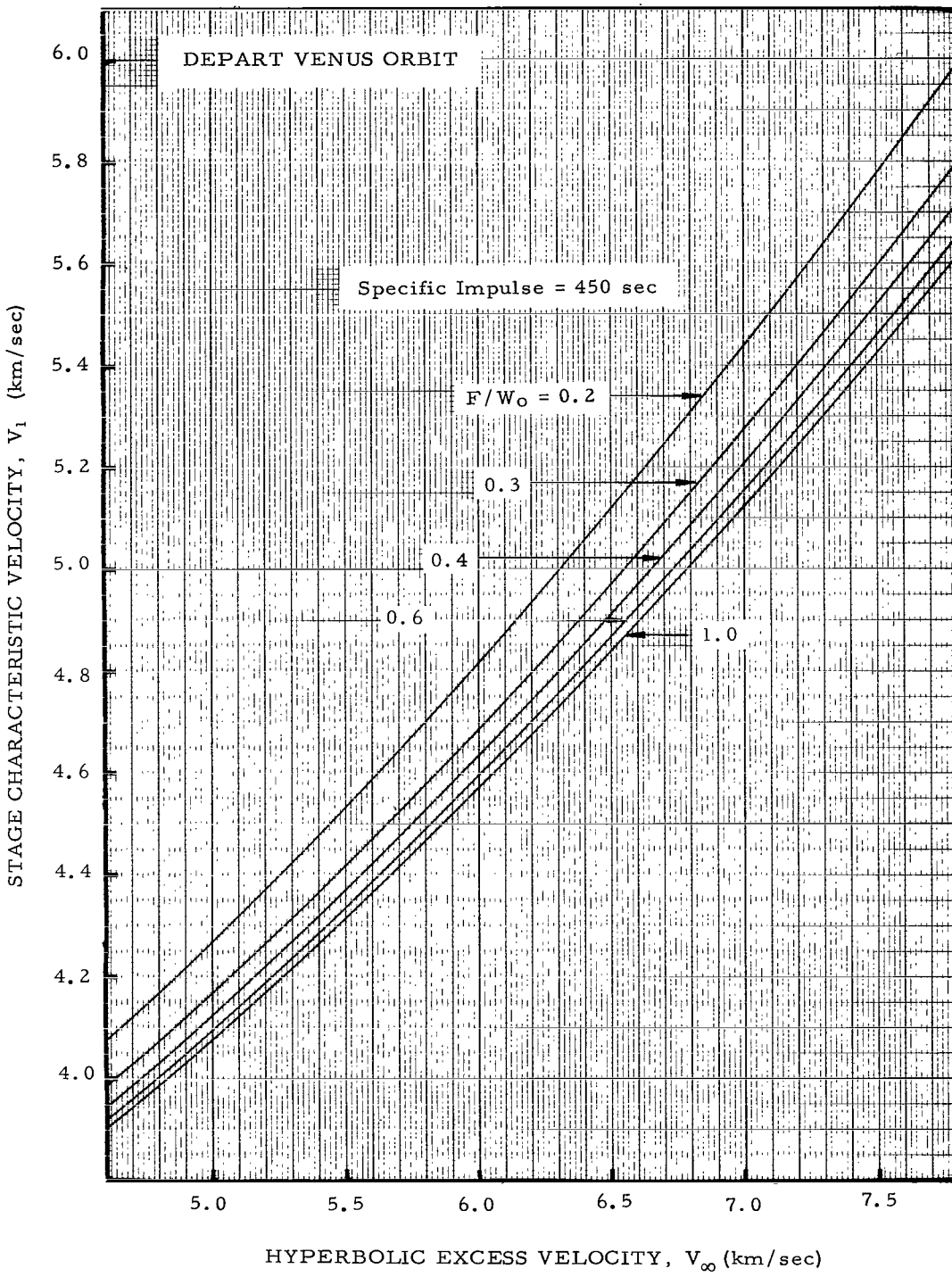
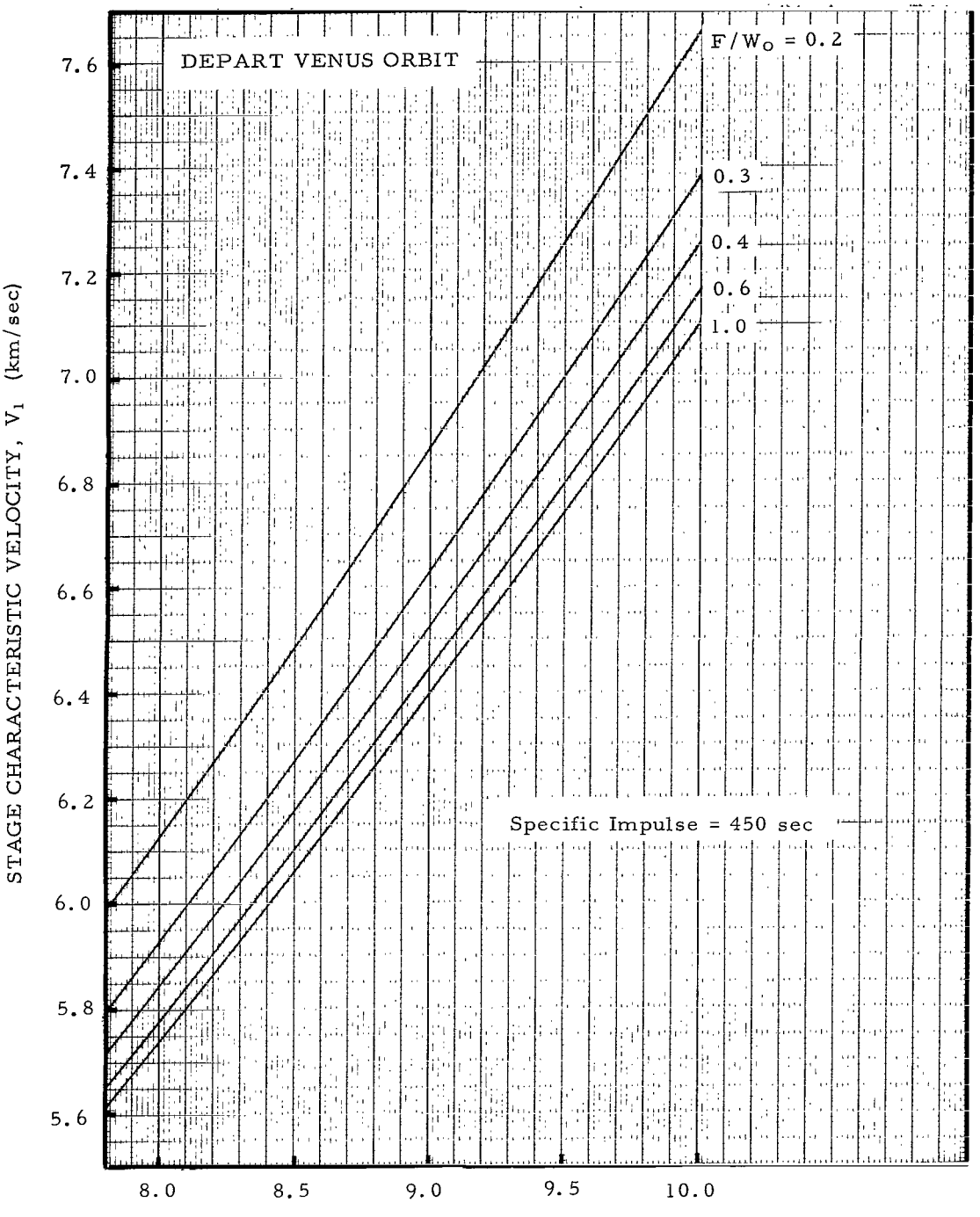


FIGURE 3b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS



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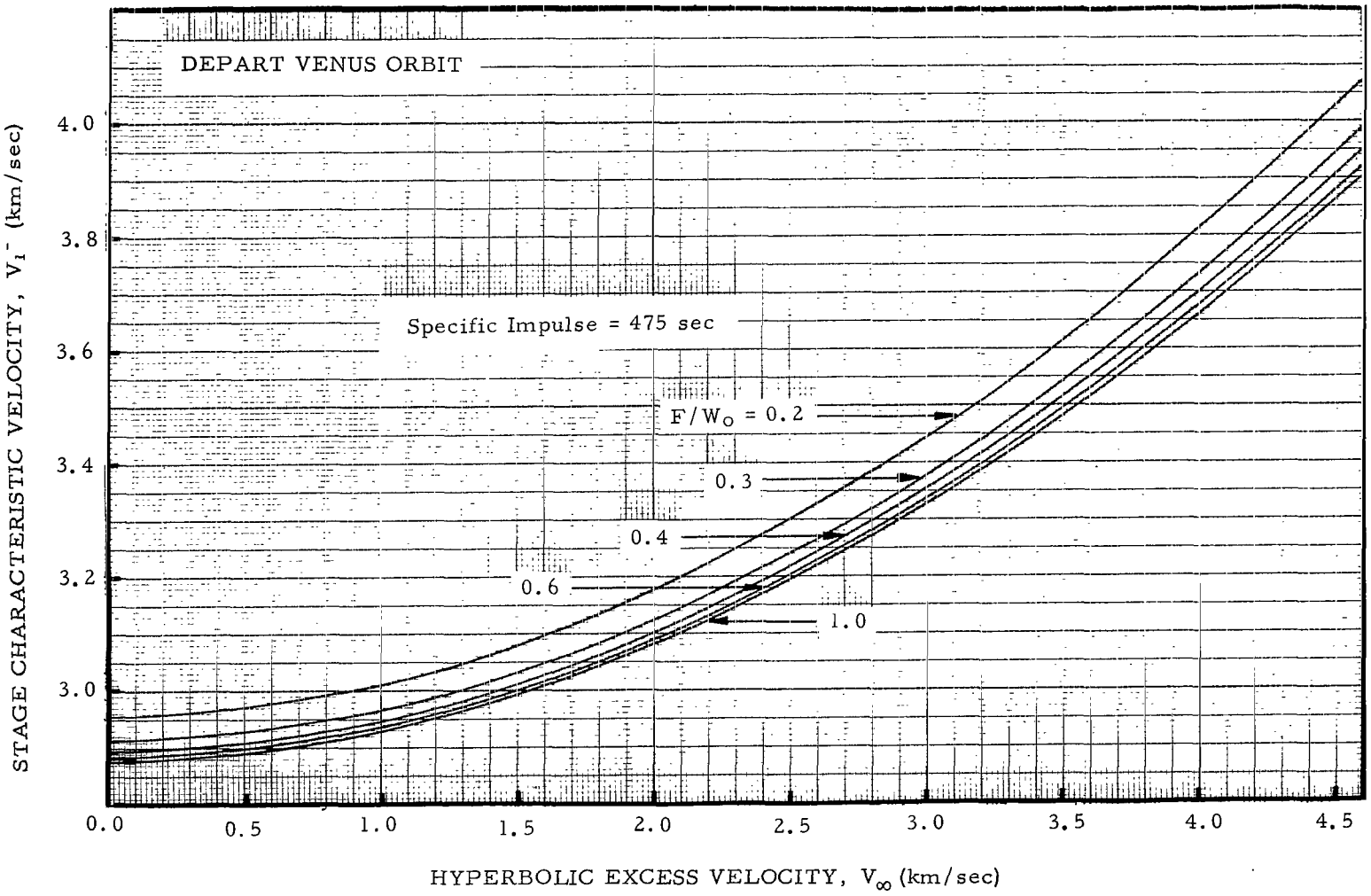


FIGURE 4a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

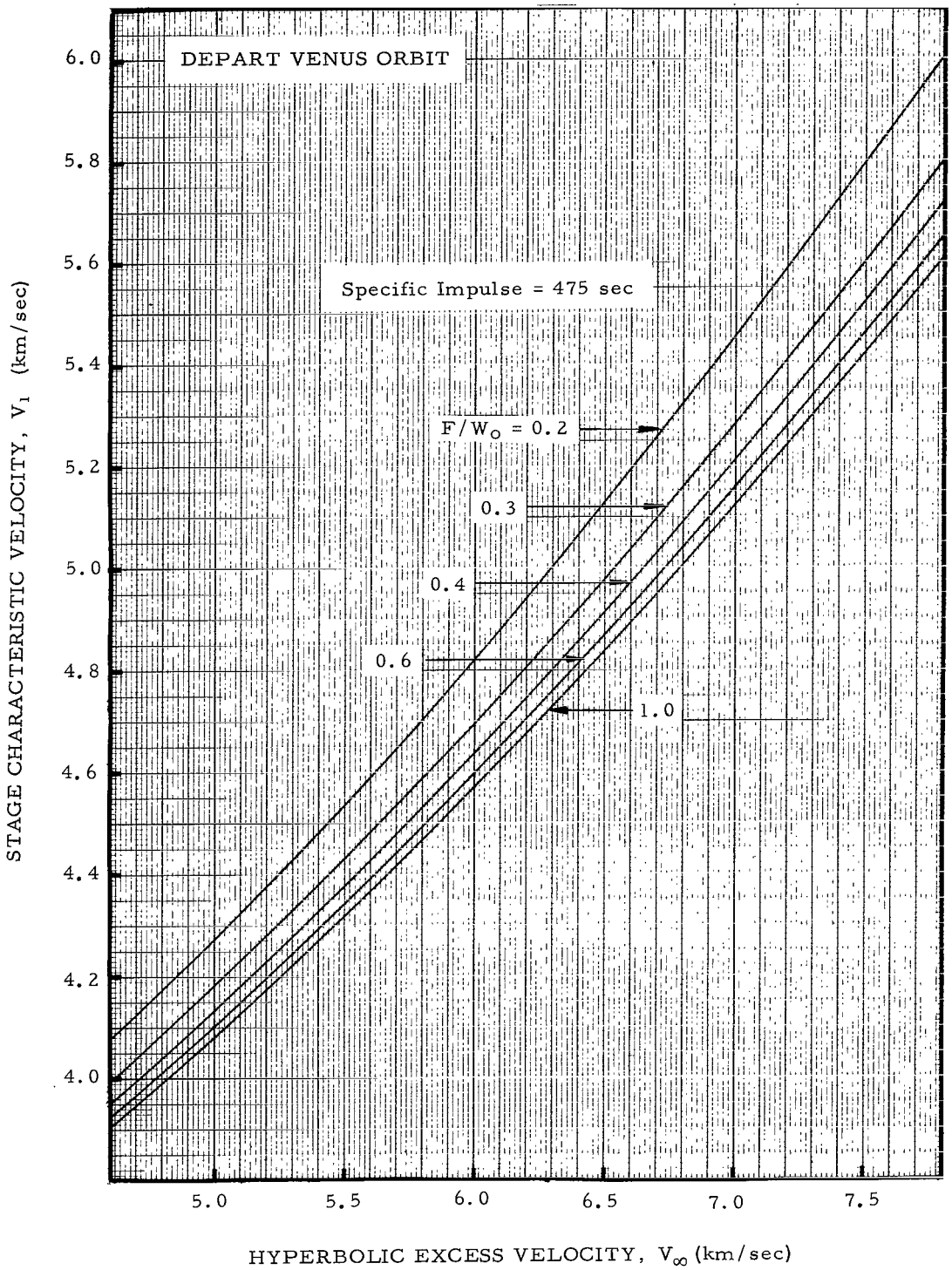


FIGURE 4b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

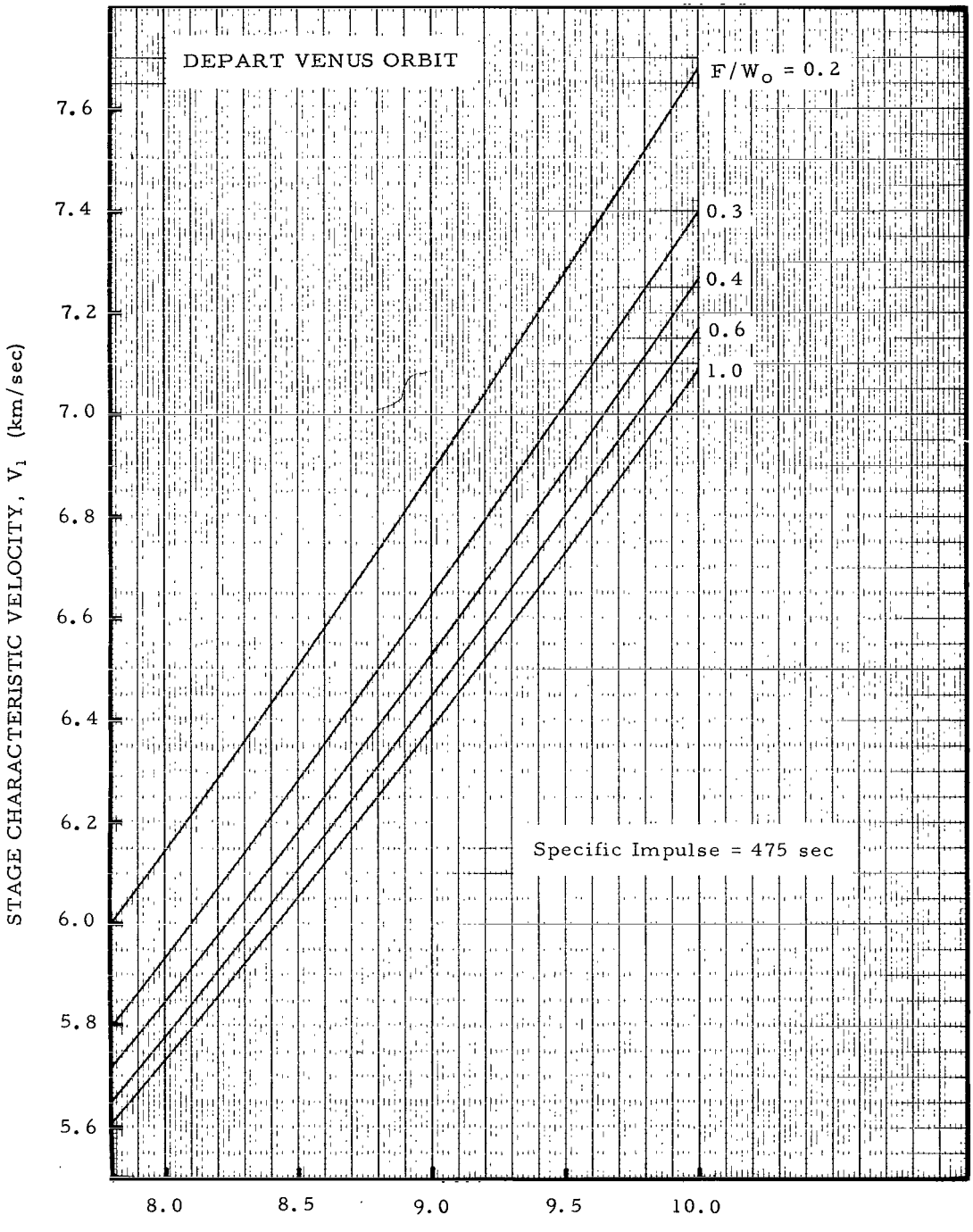


FIGURE 4c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

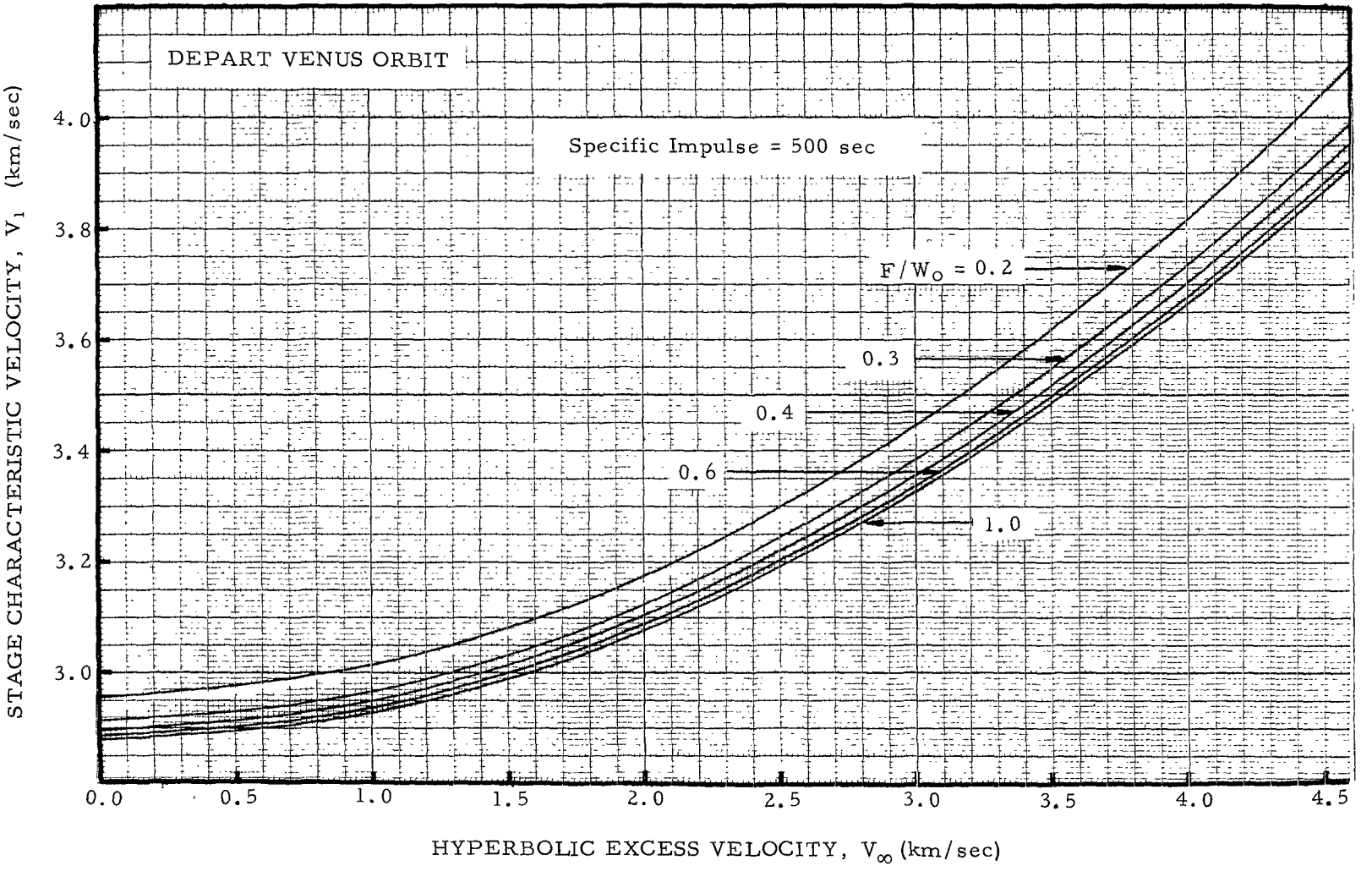


FIGURE 5a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

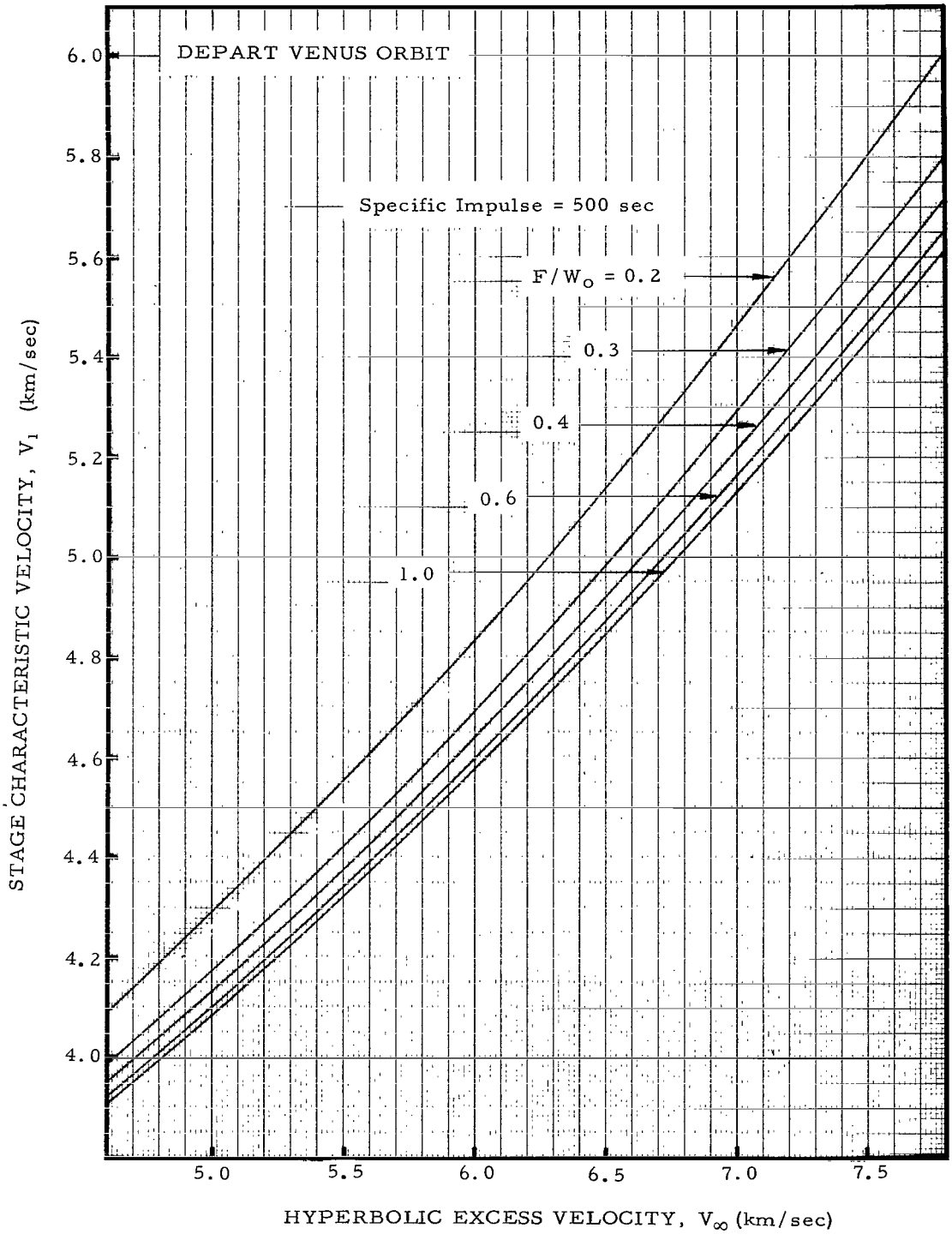


FIGURE 5b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

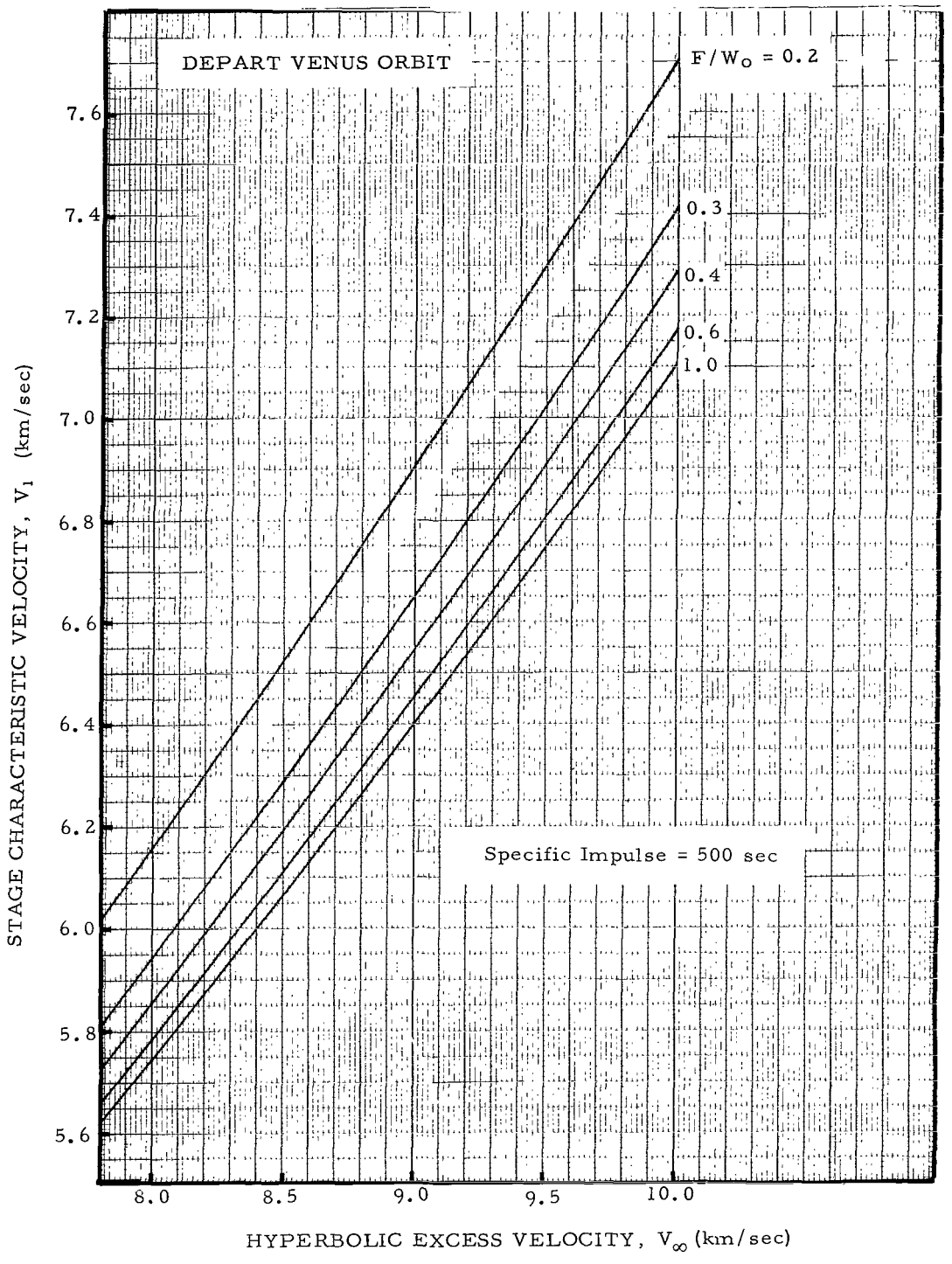


FIGURE 5c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

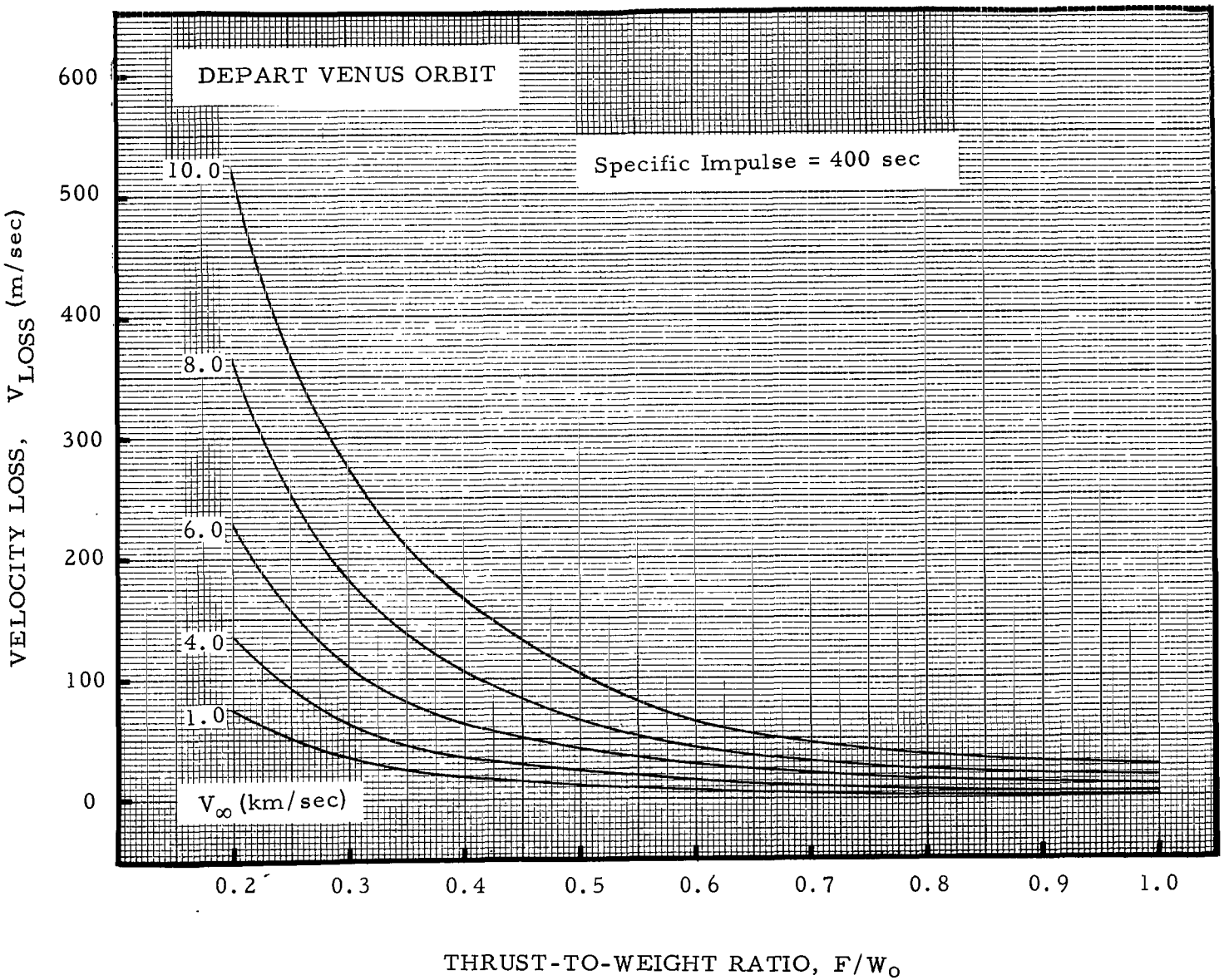


FIGURE 6. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

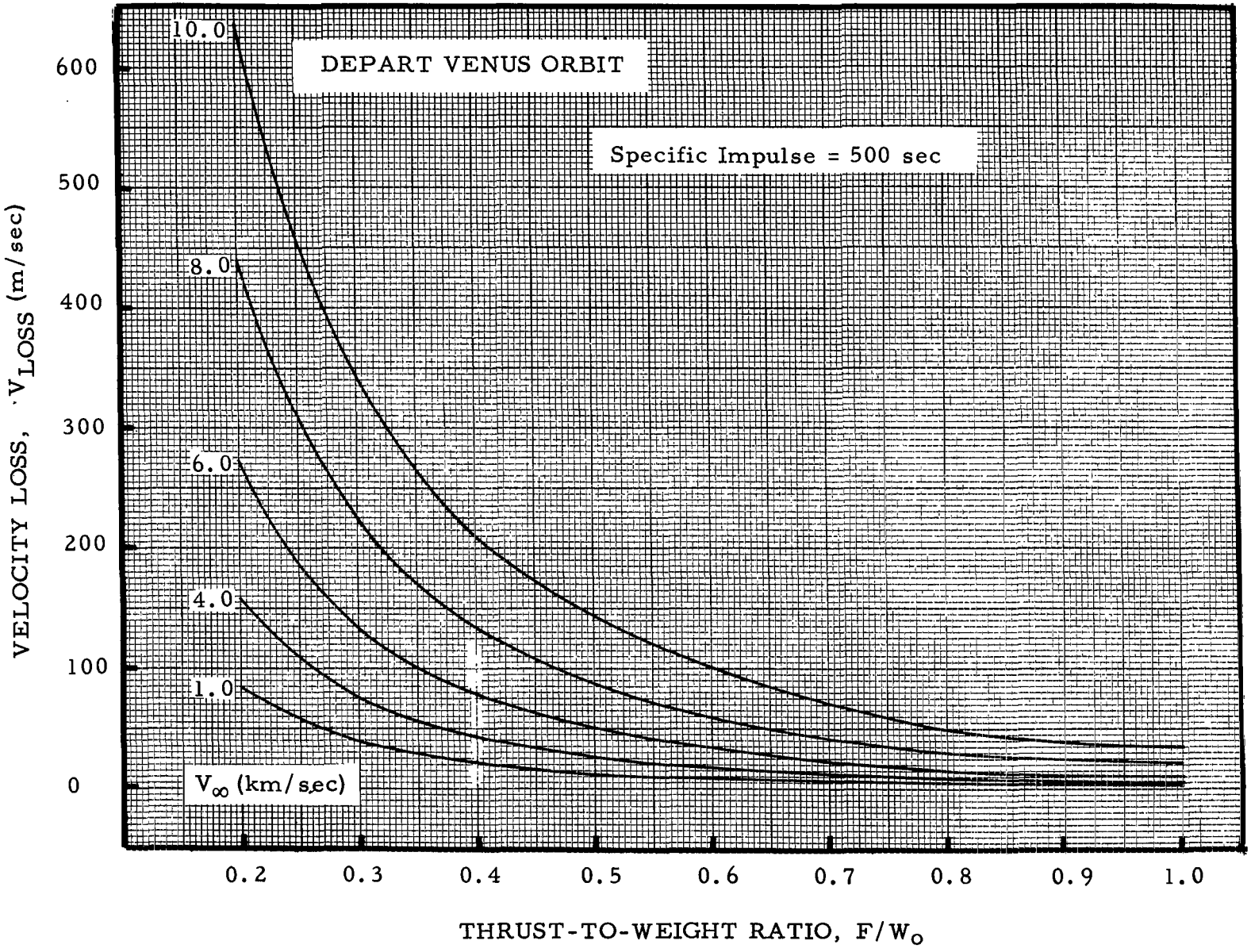


FIGURE 7. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

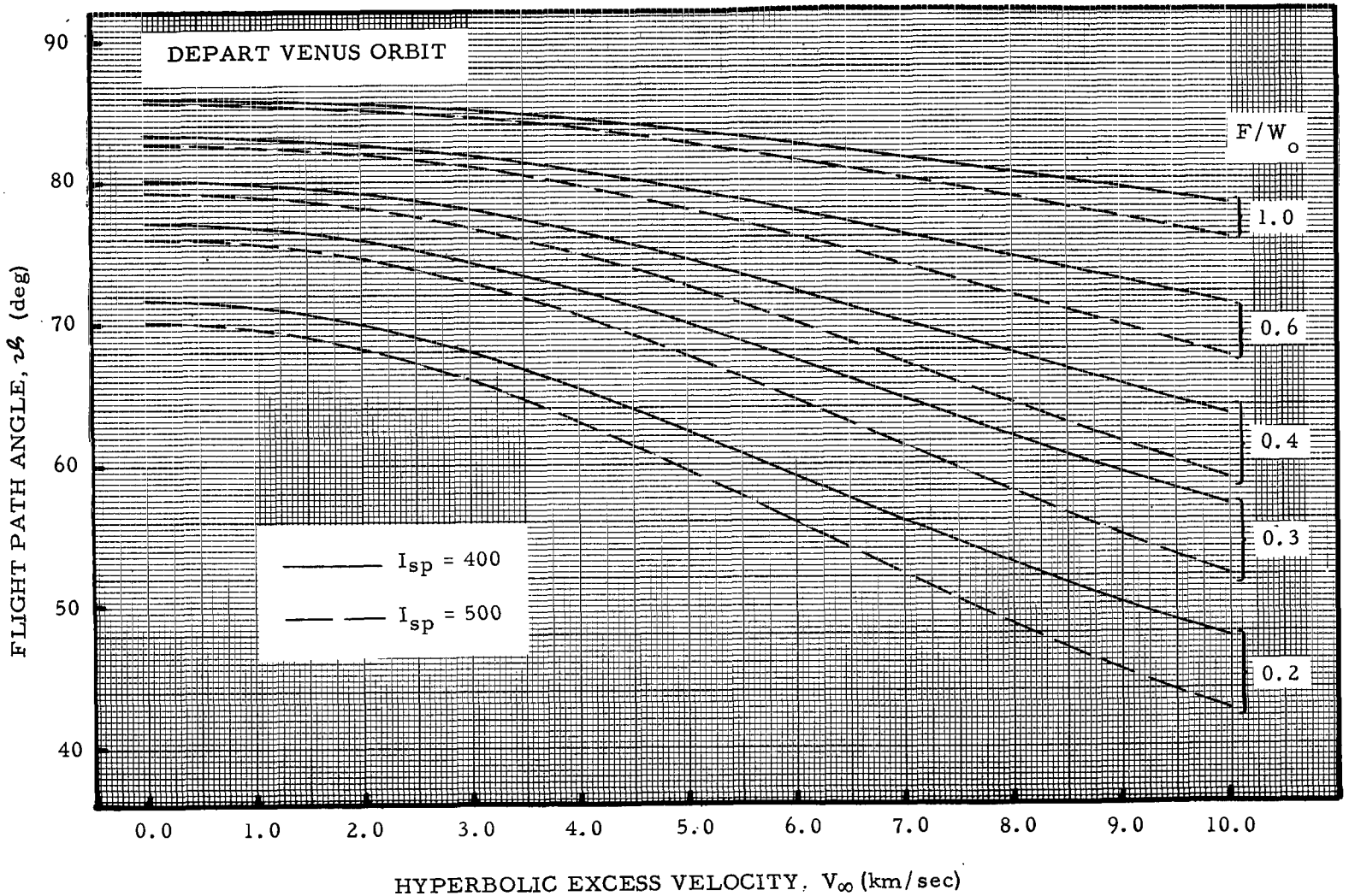


FIGURE 8. FLIGHT PATH ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

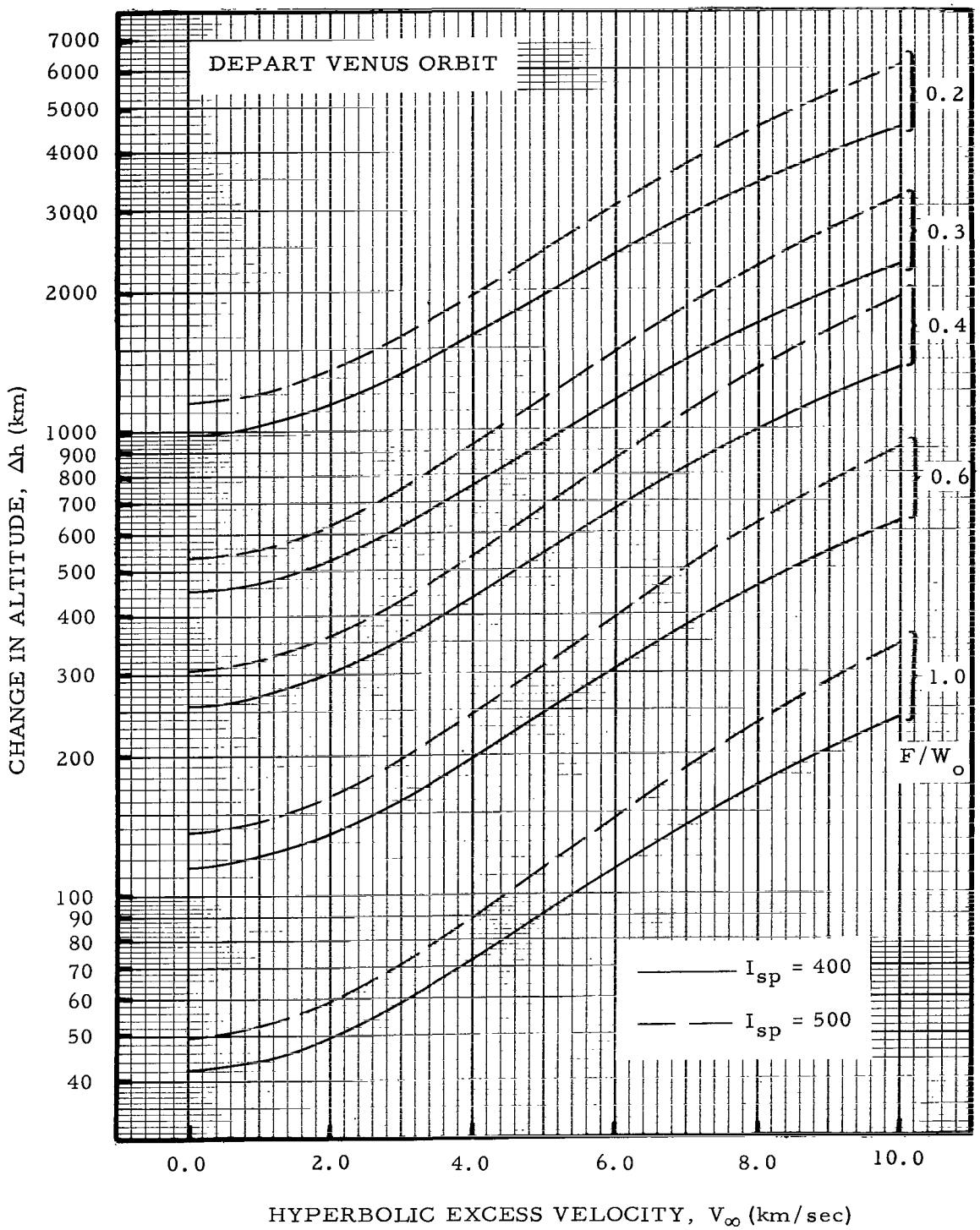


FIGURE 9. CHANGE IN ALTITUDE (km) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

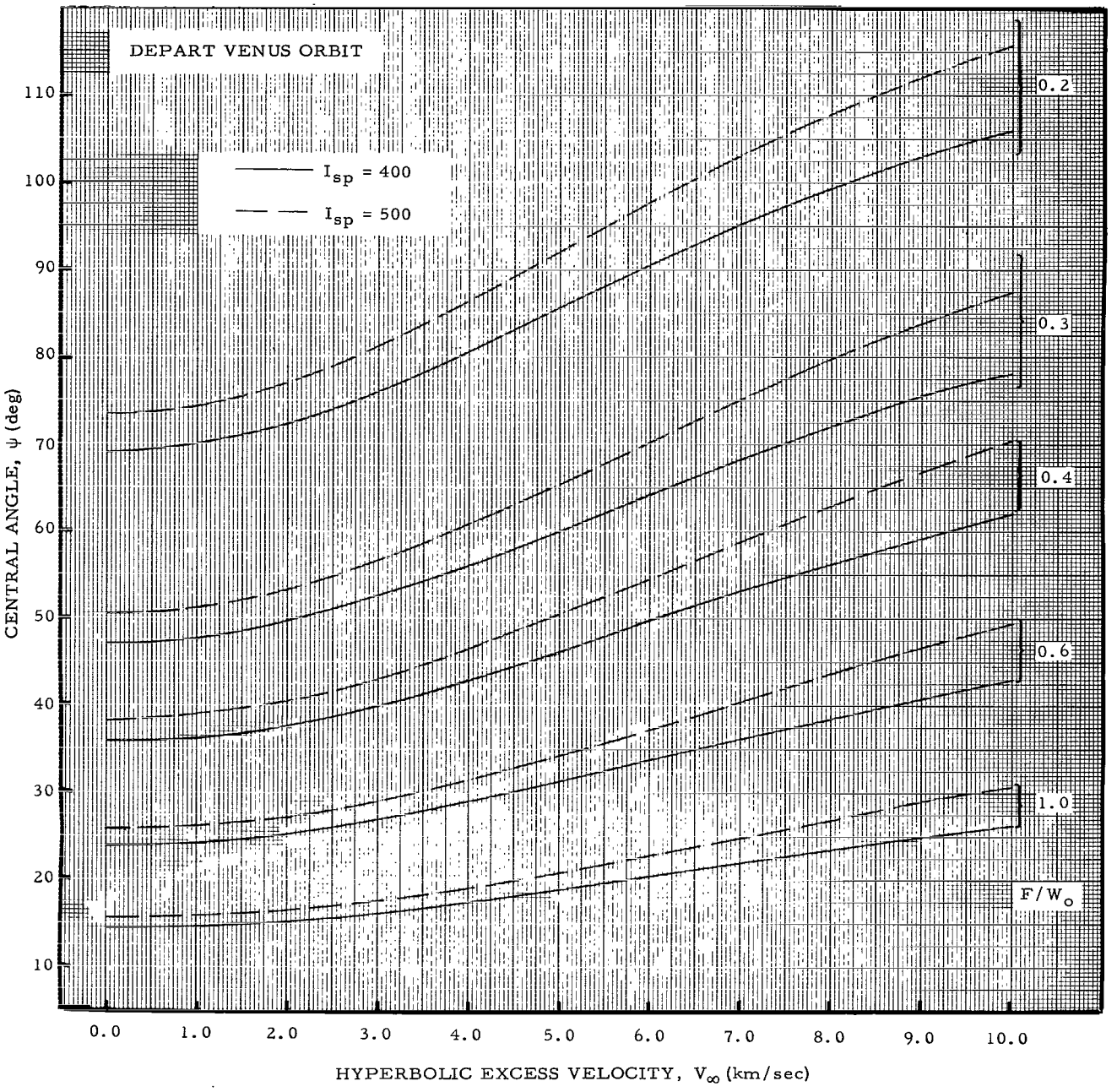


FIGURE 10. CENTRAL ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

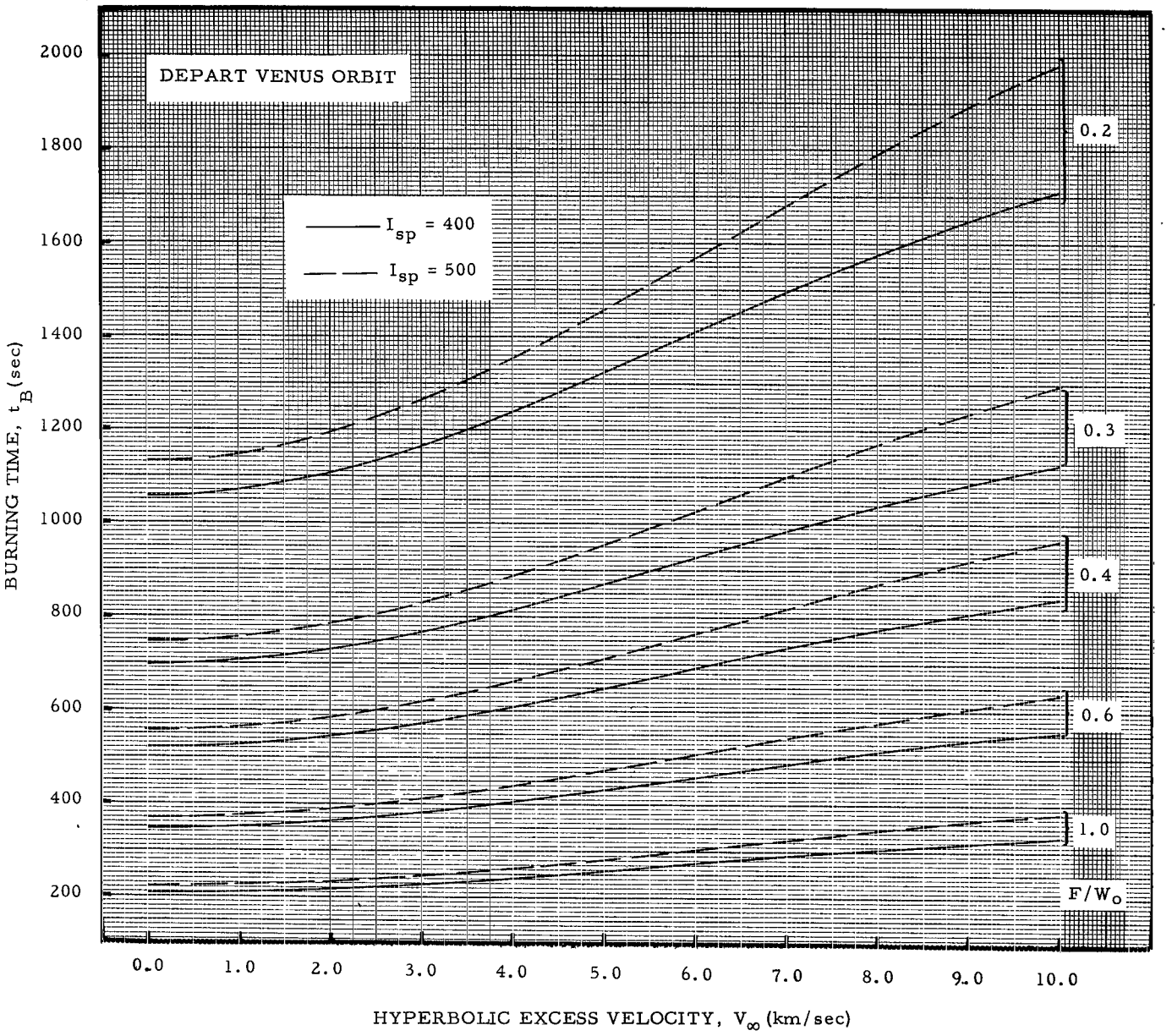


FIGURE 11. BURNING TIME (sec) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

DEPARTURE FROM MARS ORBIT

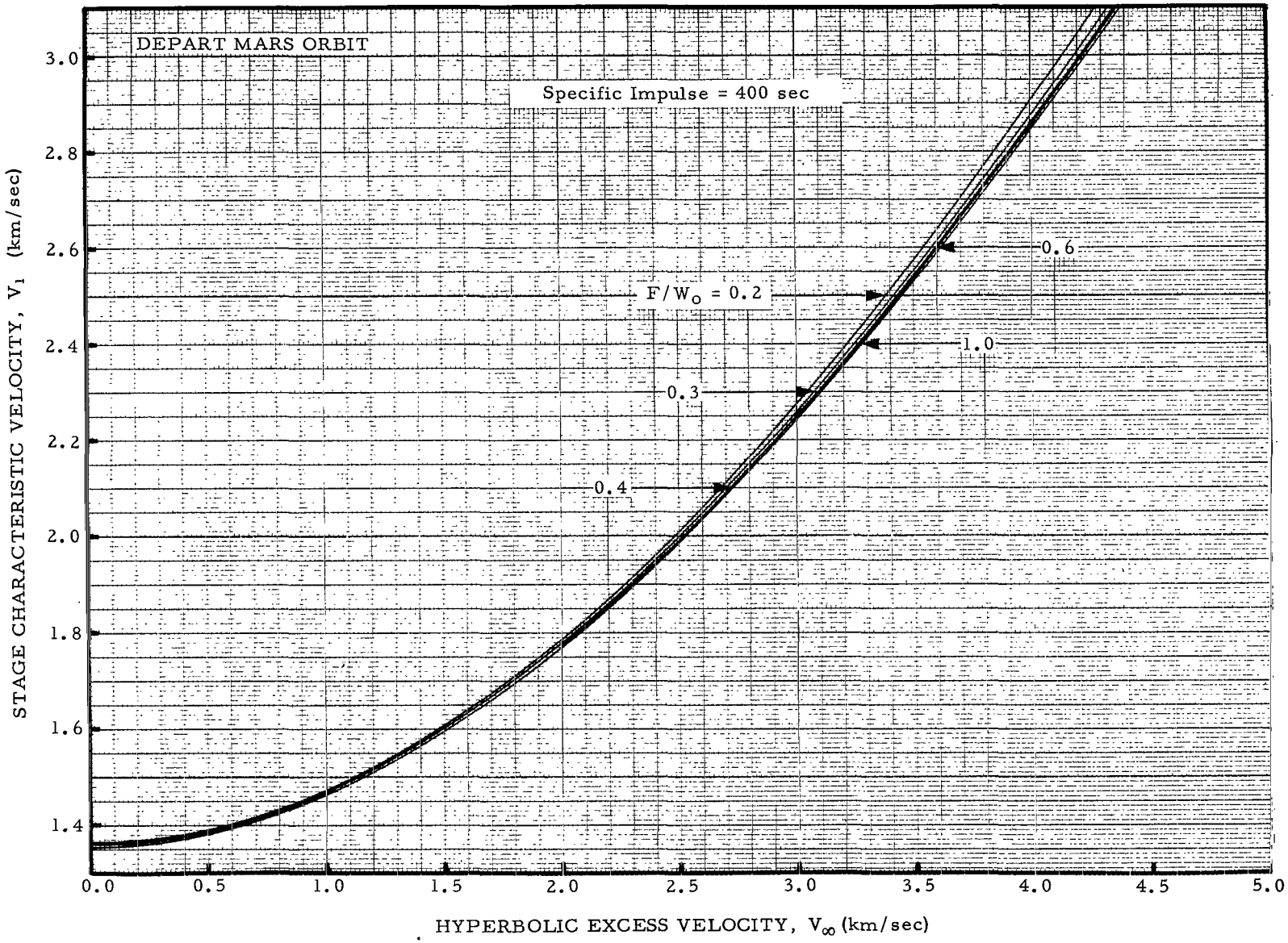


FIGURE 1a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

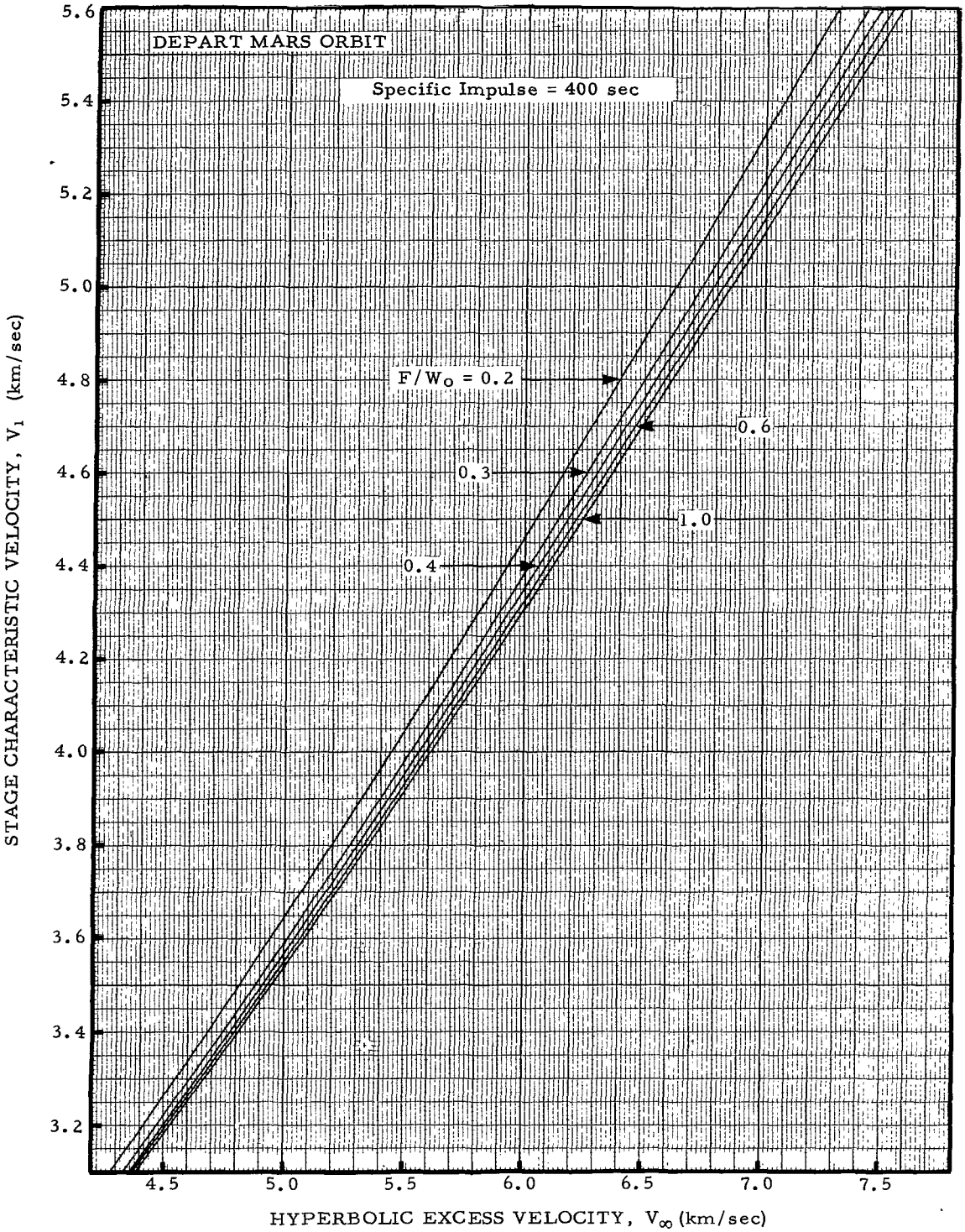


FIGURE 1b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

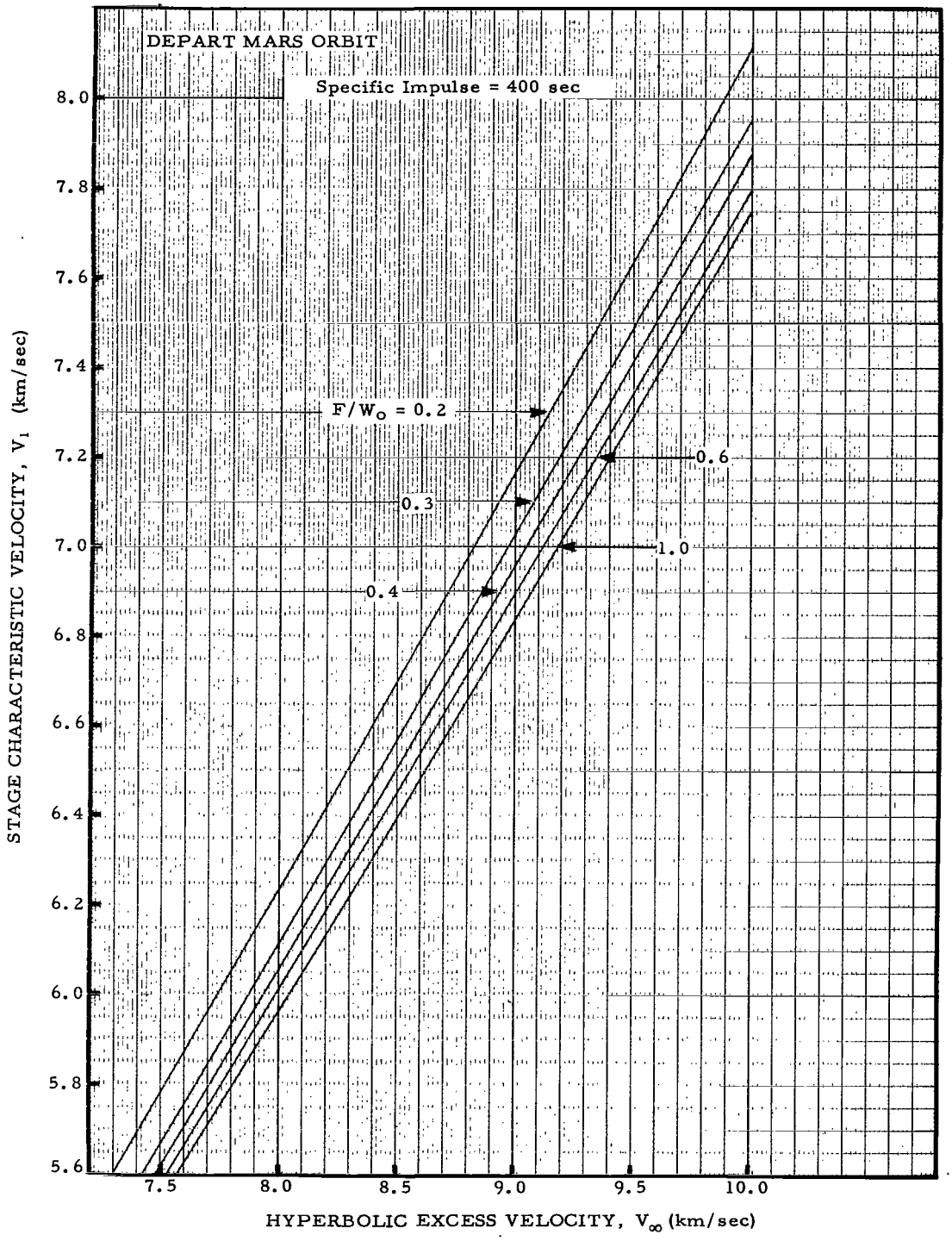


FIGURE 1c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

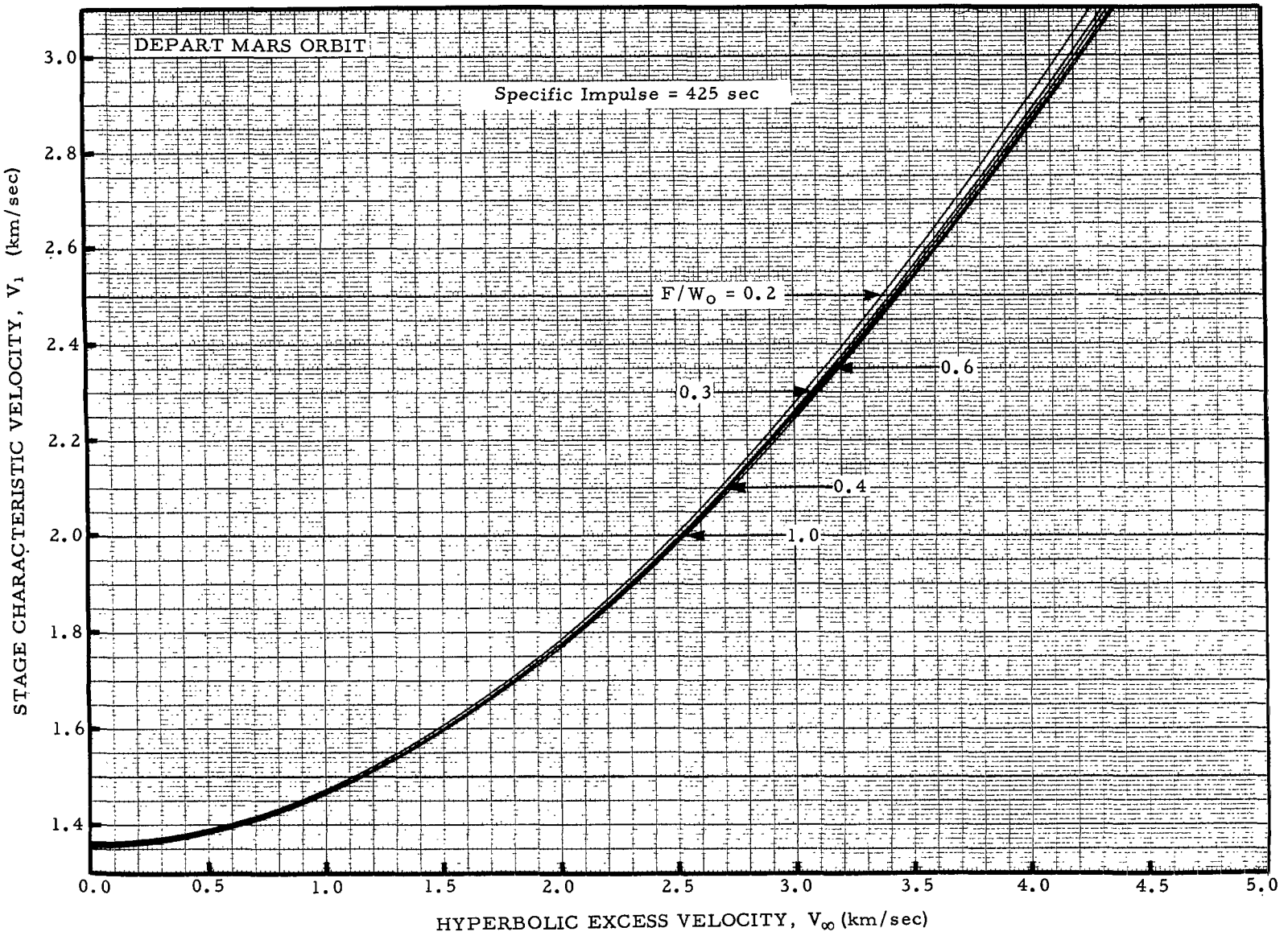


FIGURE 2a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

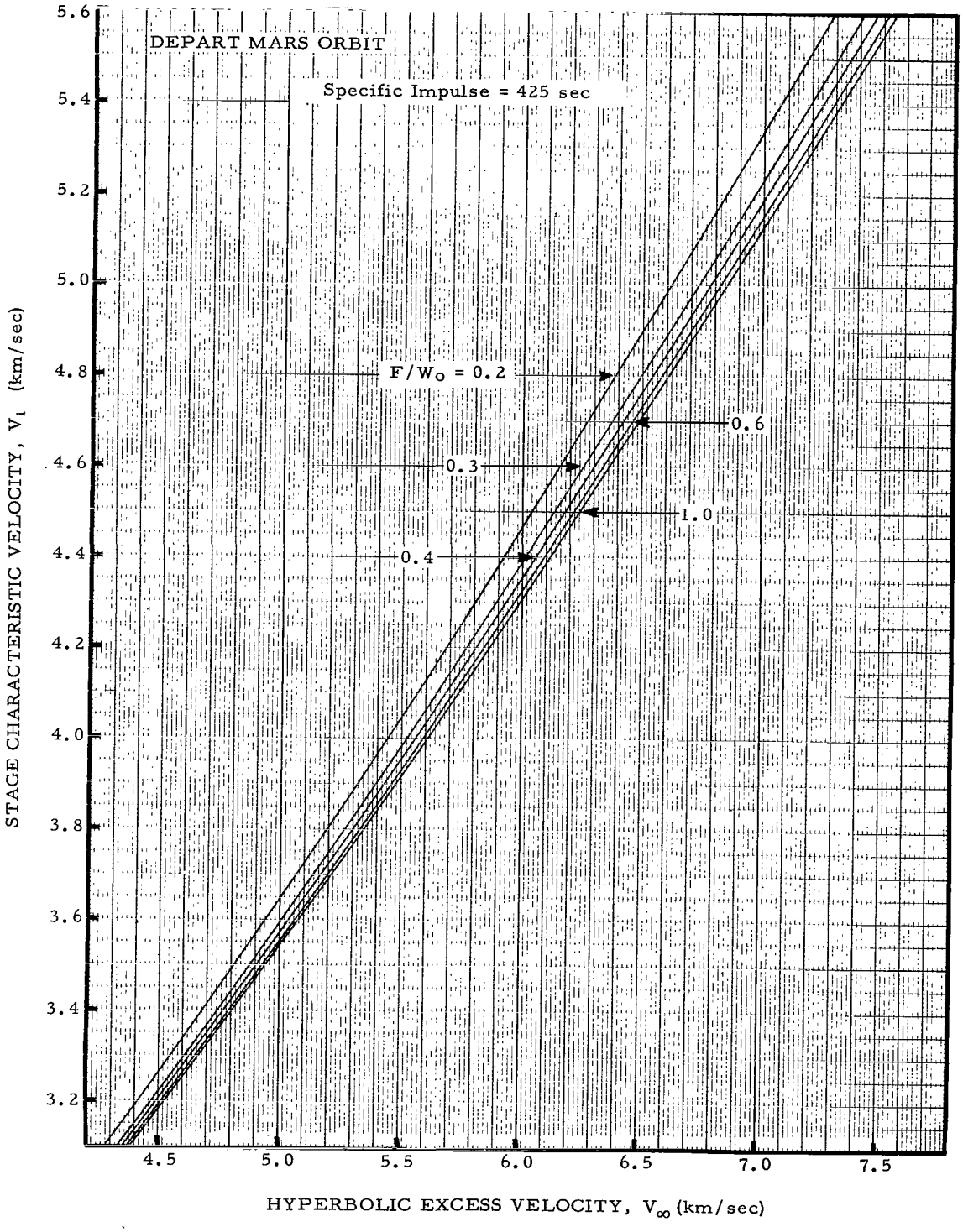


FIGURE 2b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

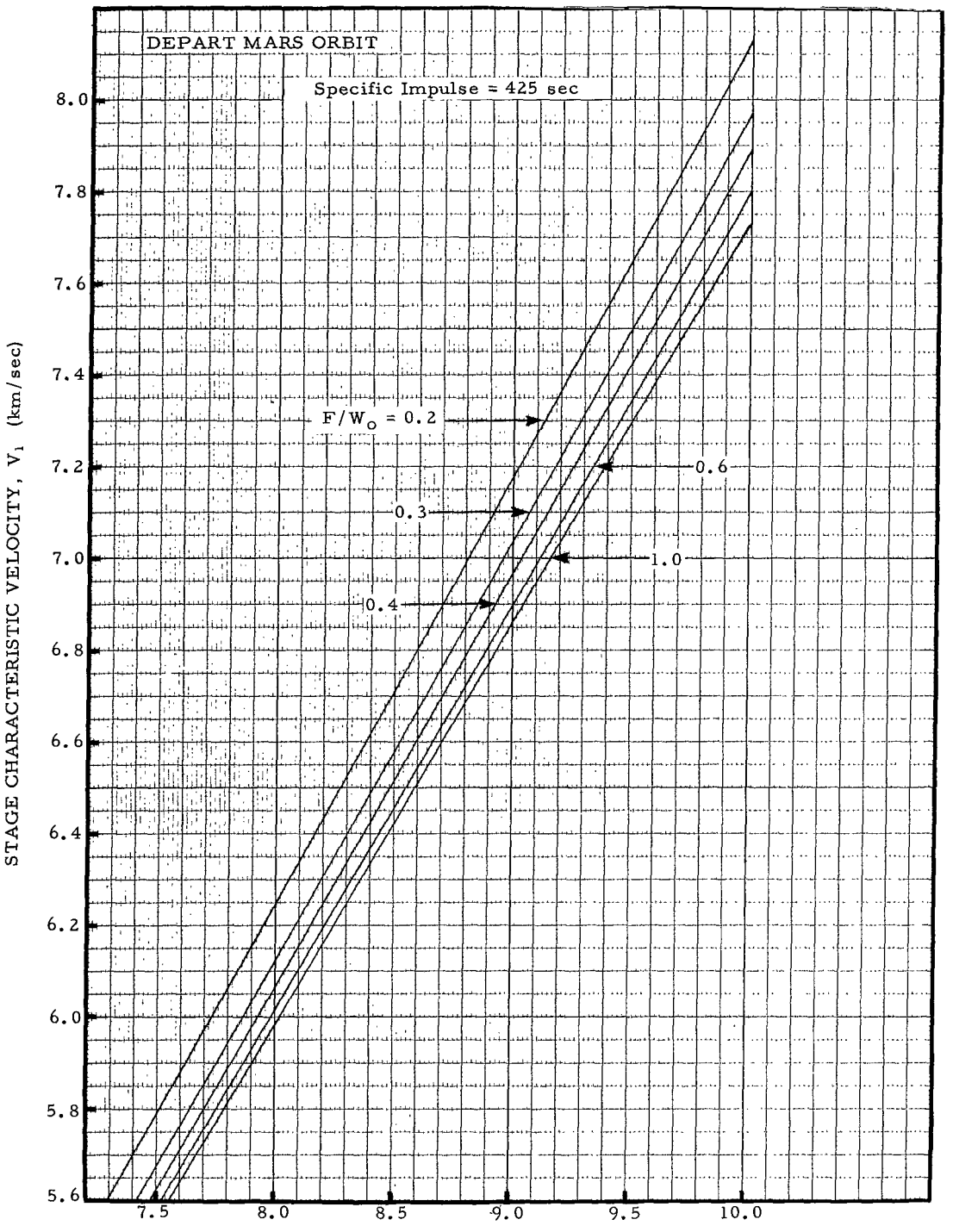


FIGURE 2c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

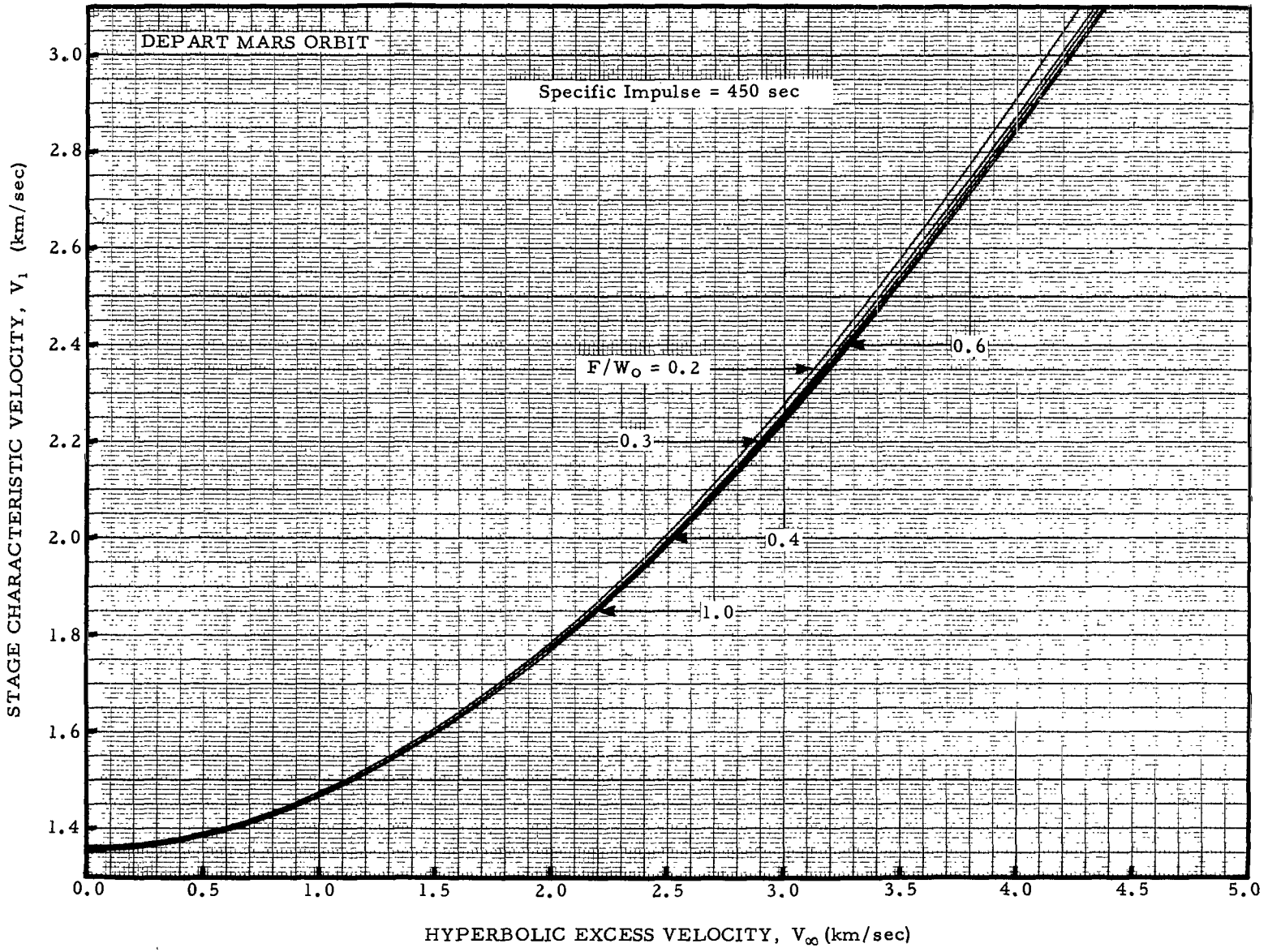


FIGURE 3a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

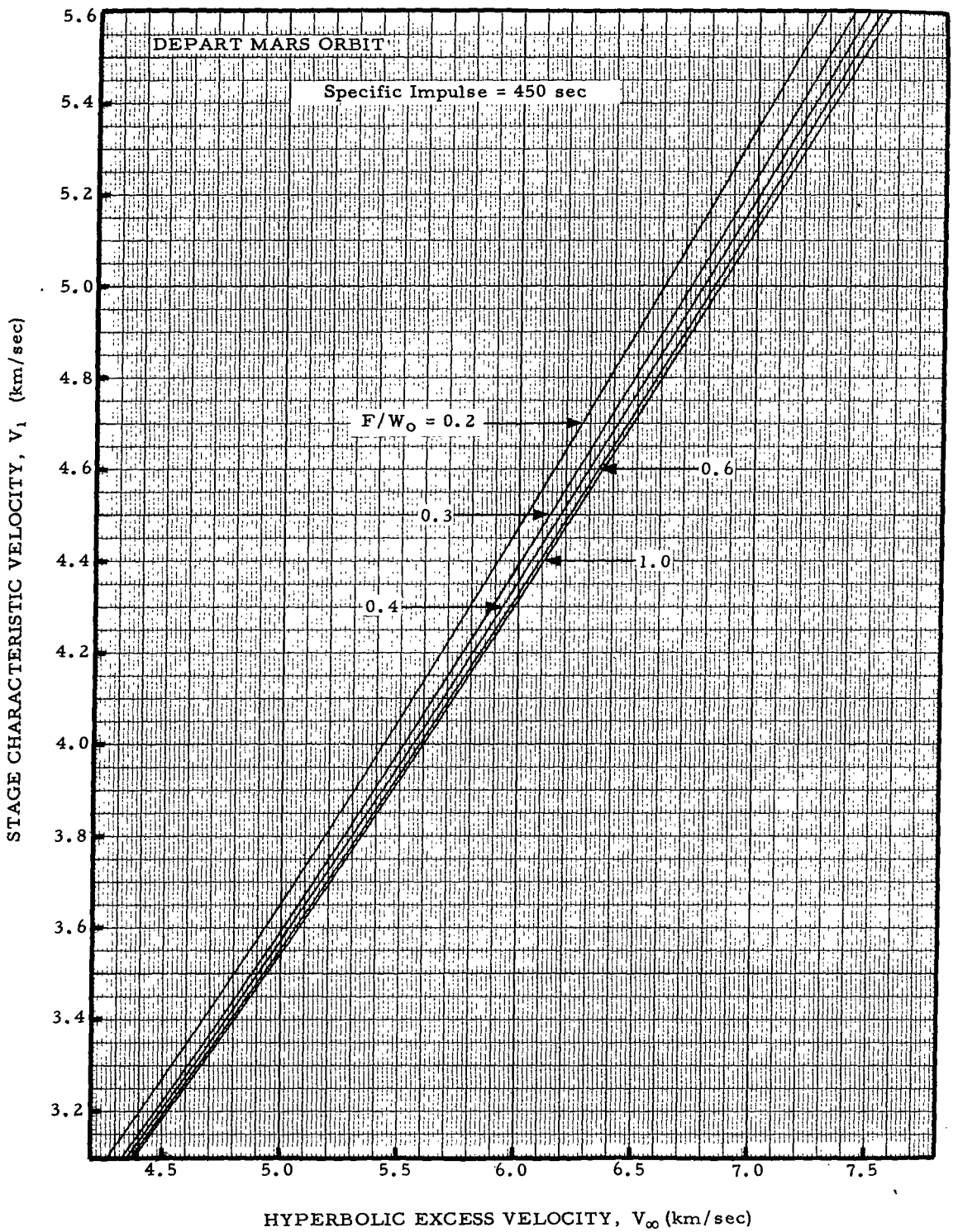


FIGURE 3b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

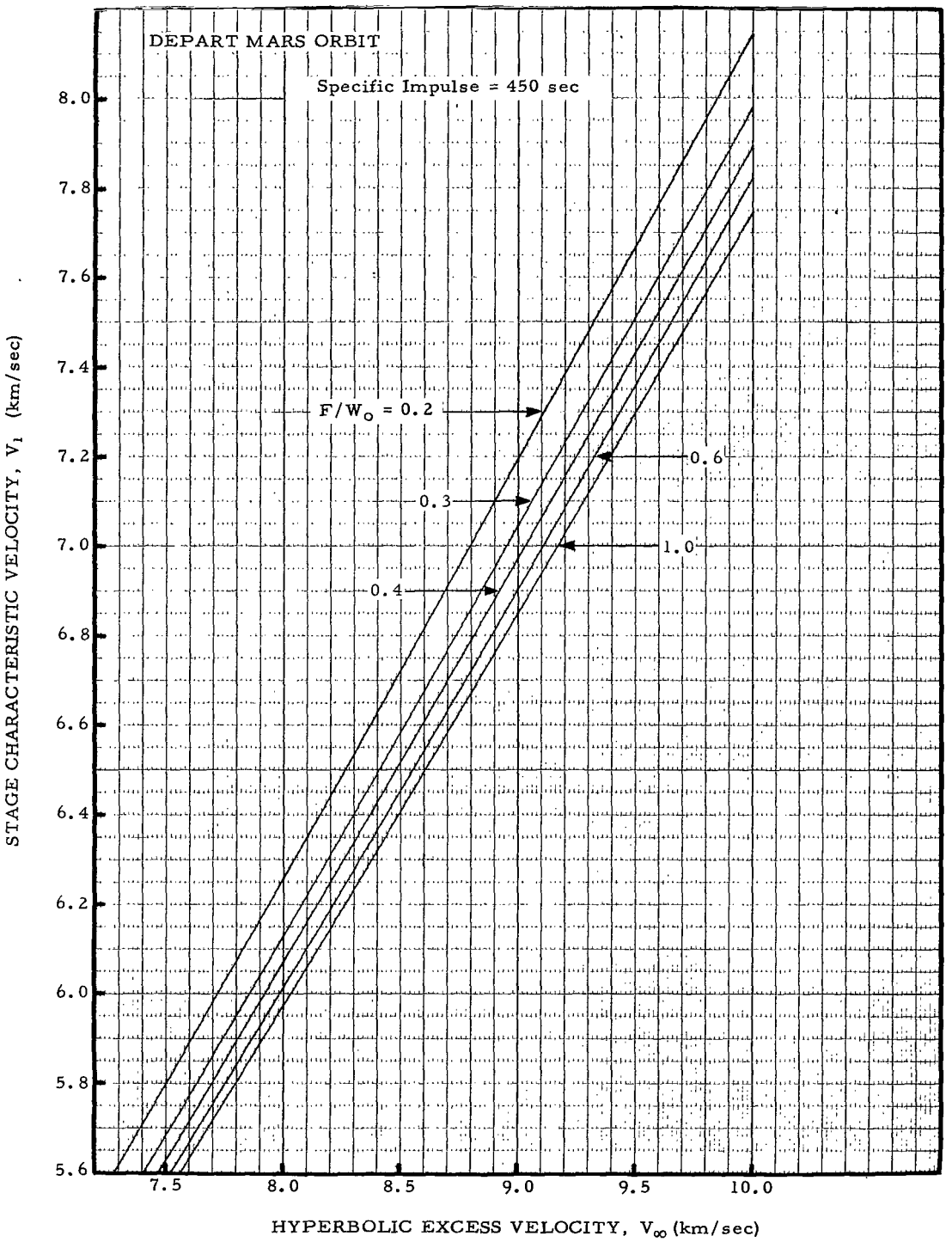


FIGURE 3c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

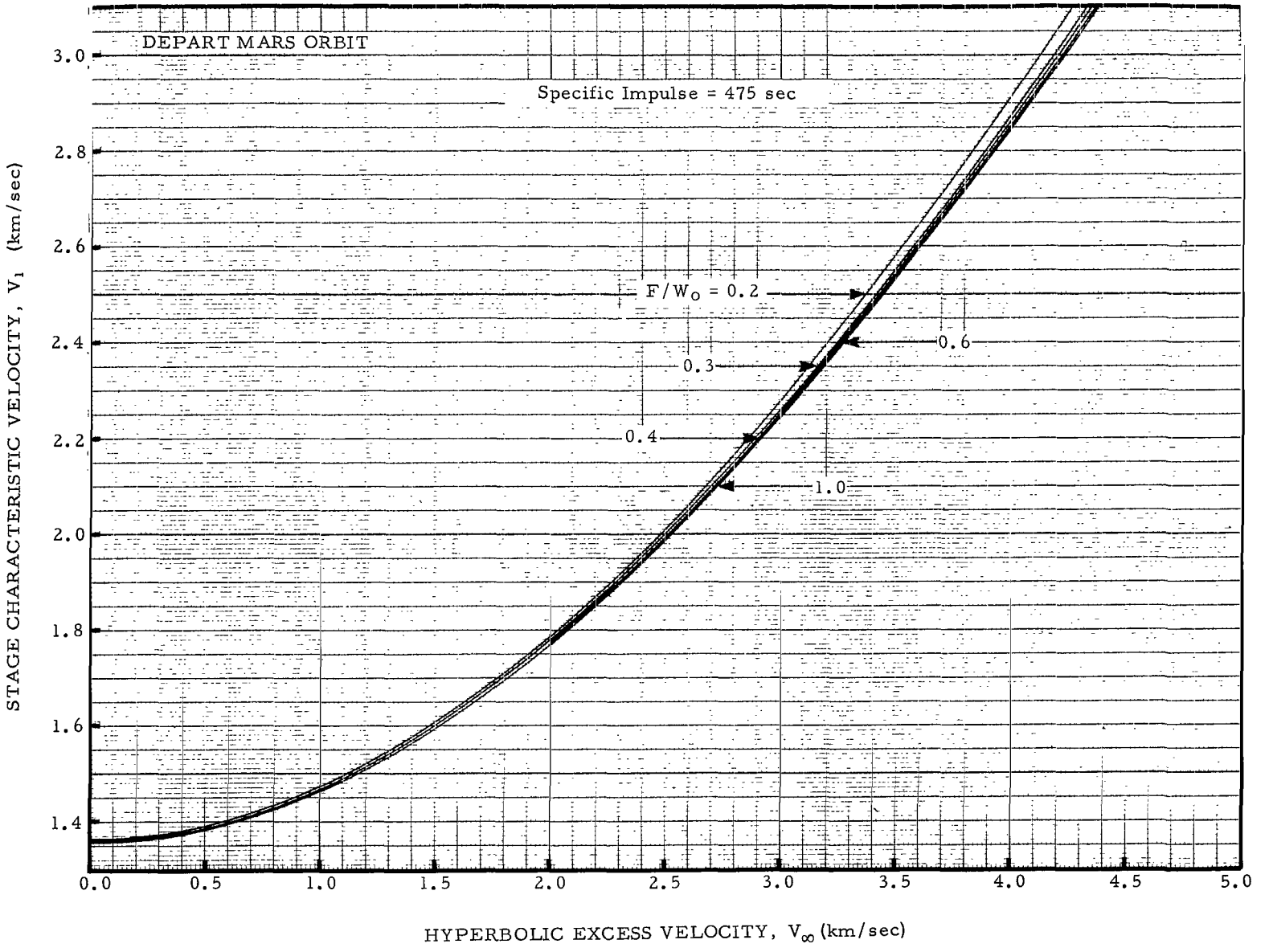


FIGURE 4a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

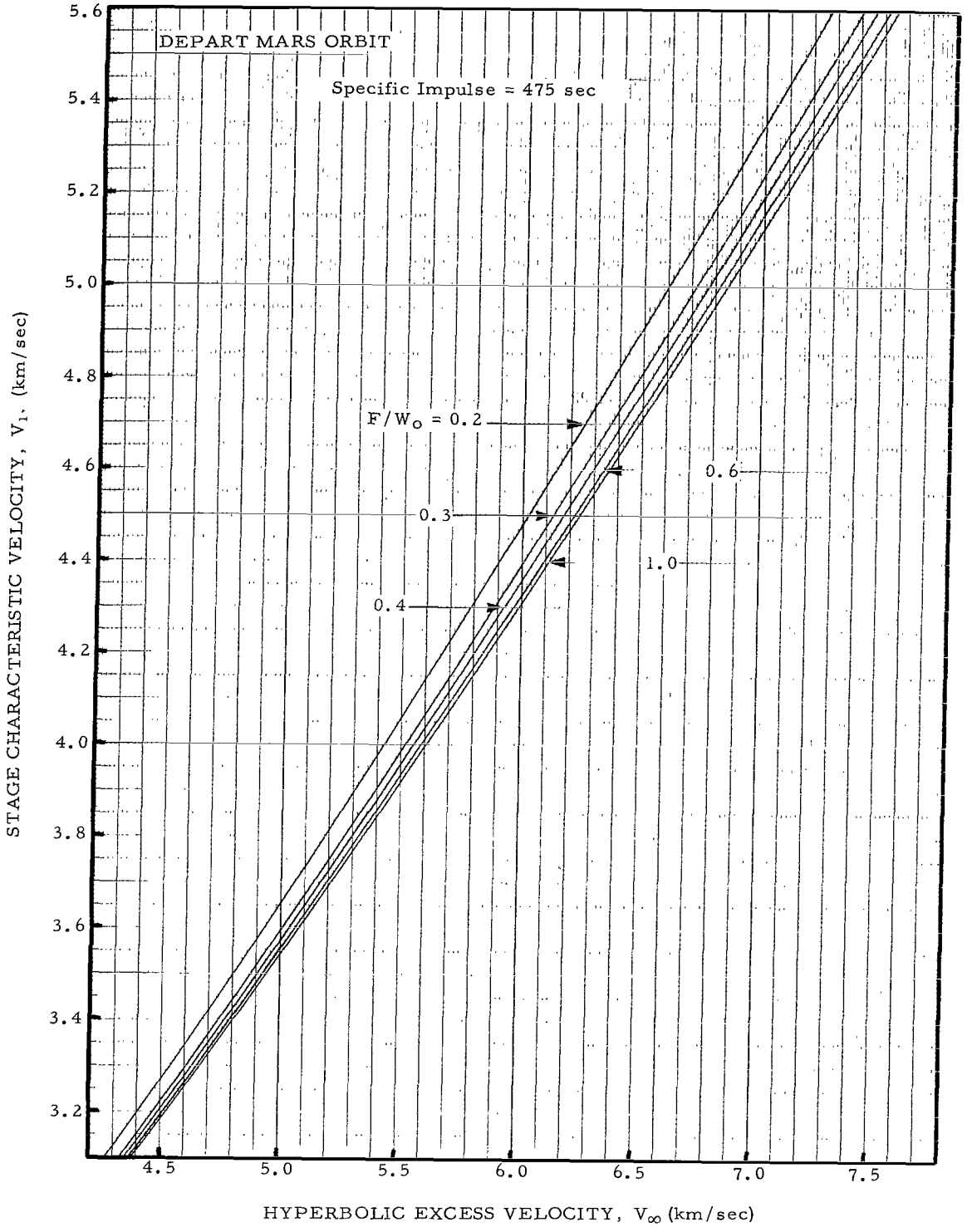


FIGURE 4b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

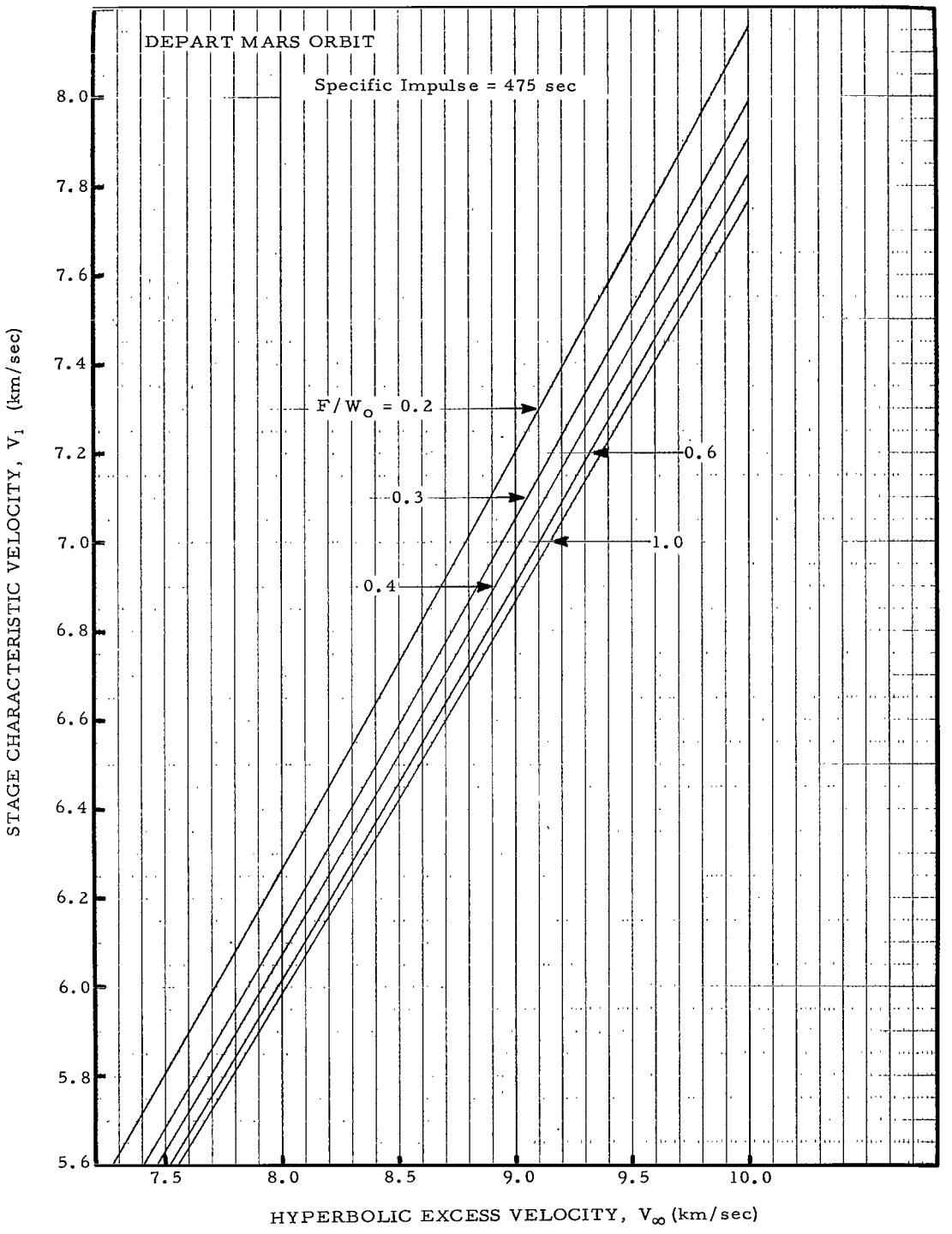


FIGURE 4c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

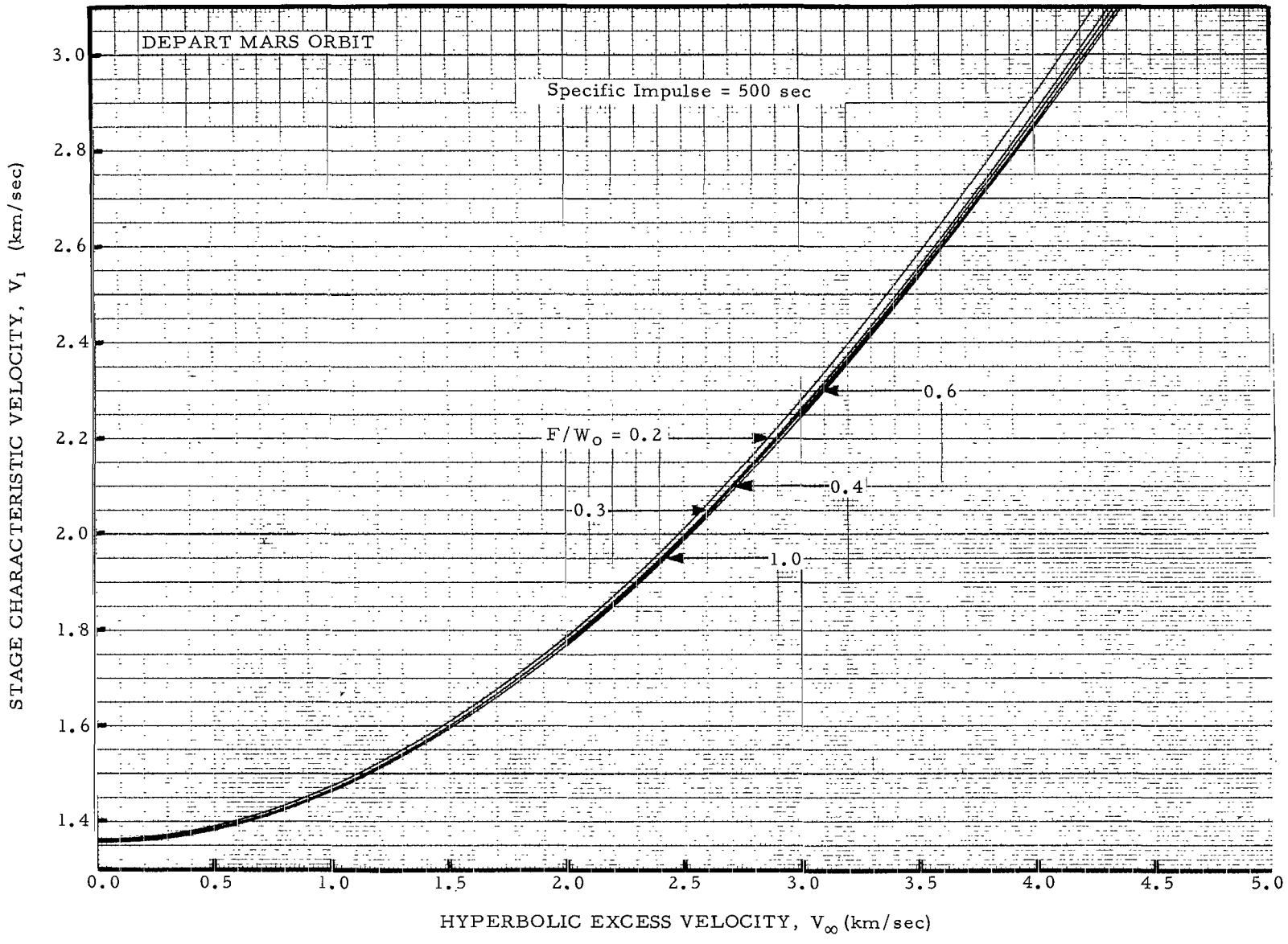


FIGURE 5a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

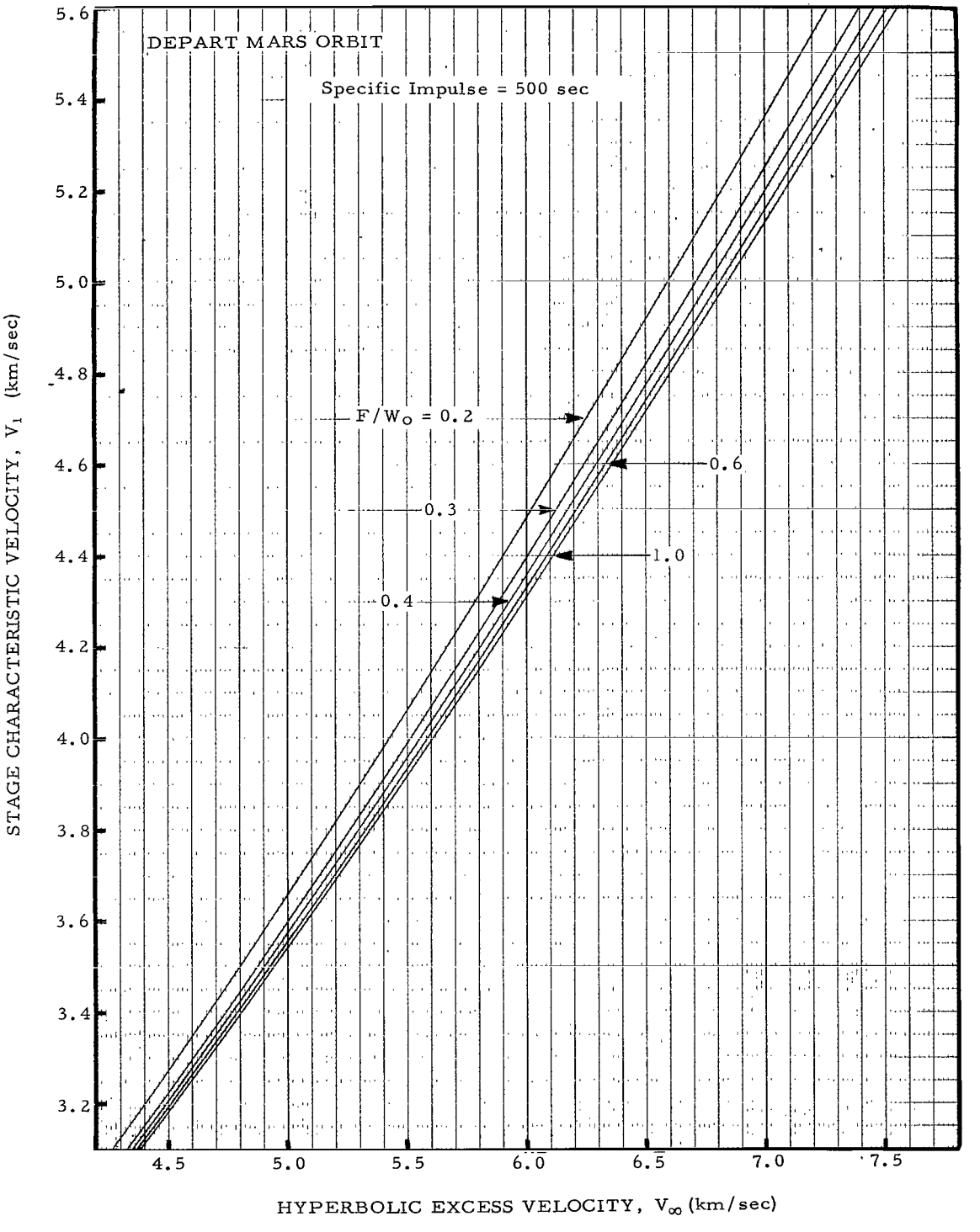


FIGURE 5b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

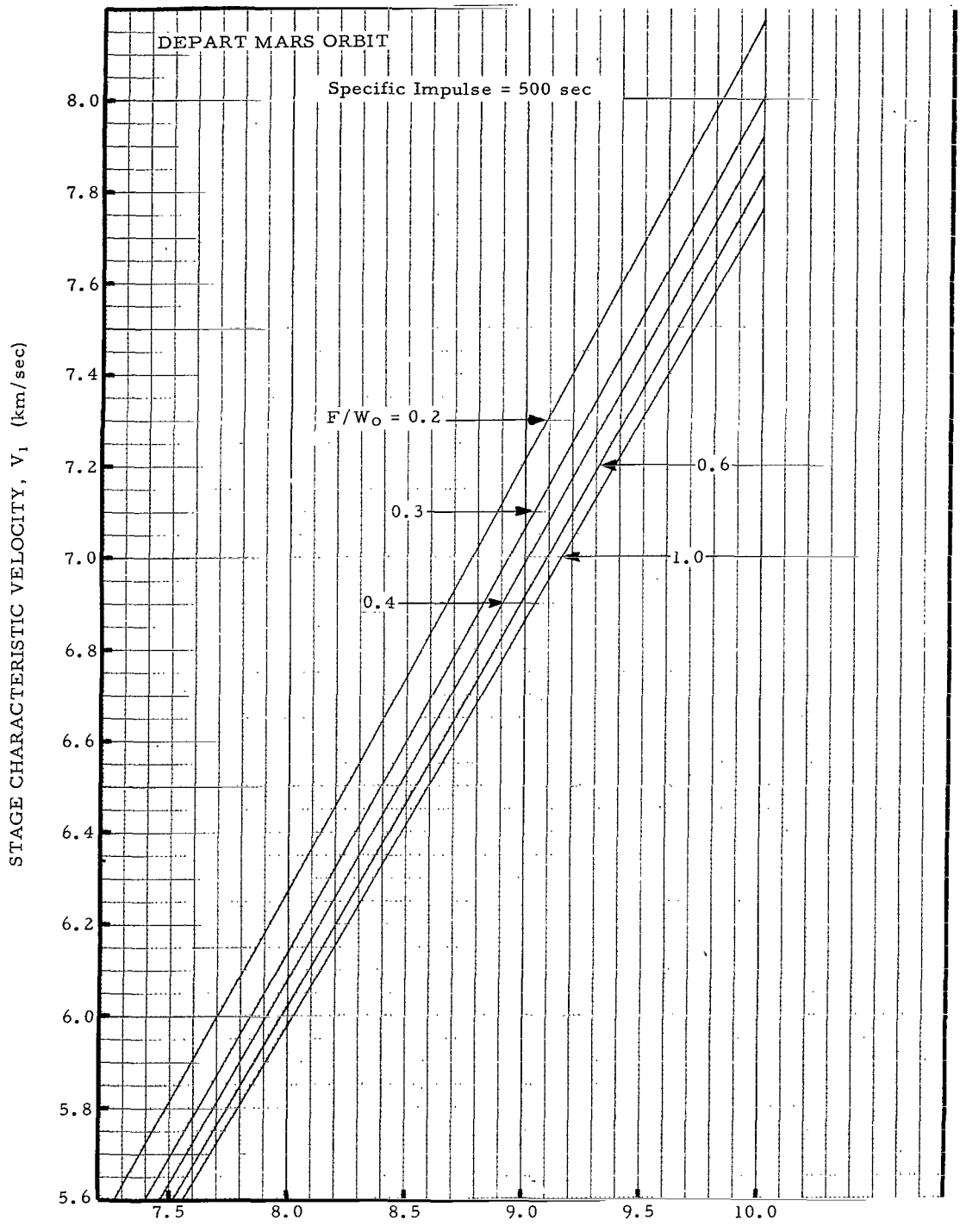


FIGURE 5c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

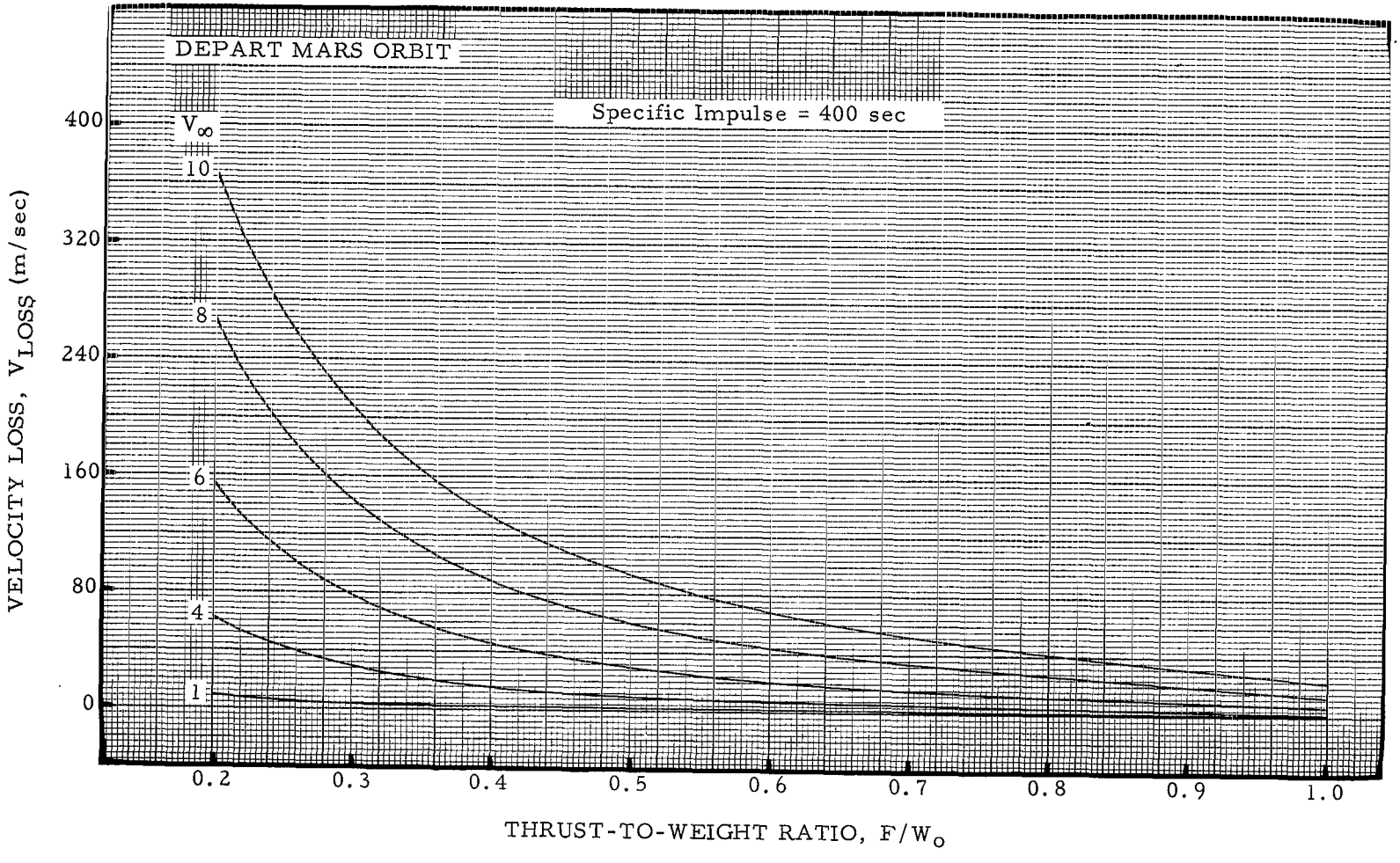


FIGURE 6. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

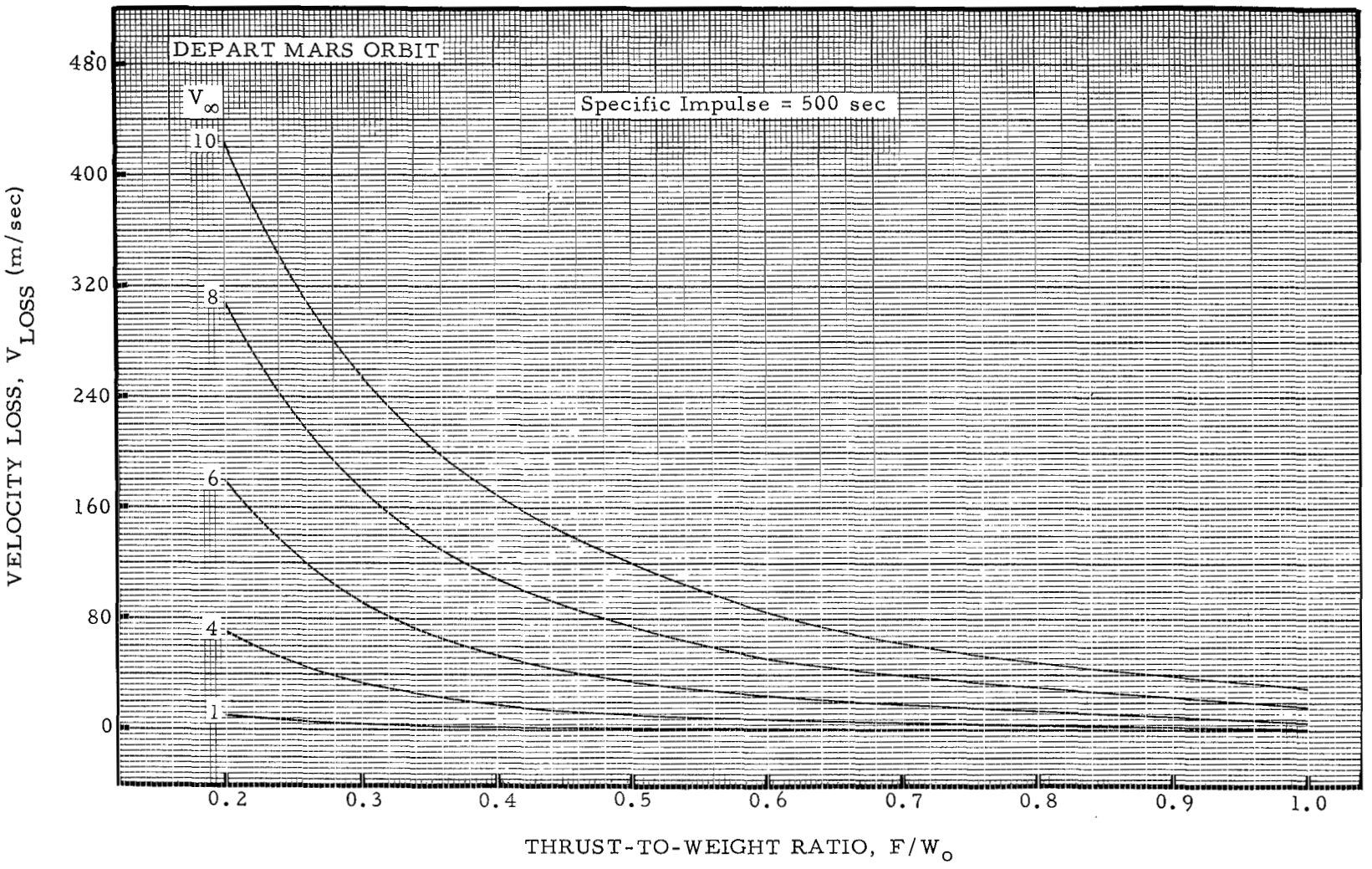


FIGURE 7. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

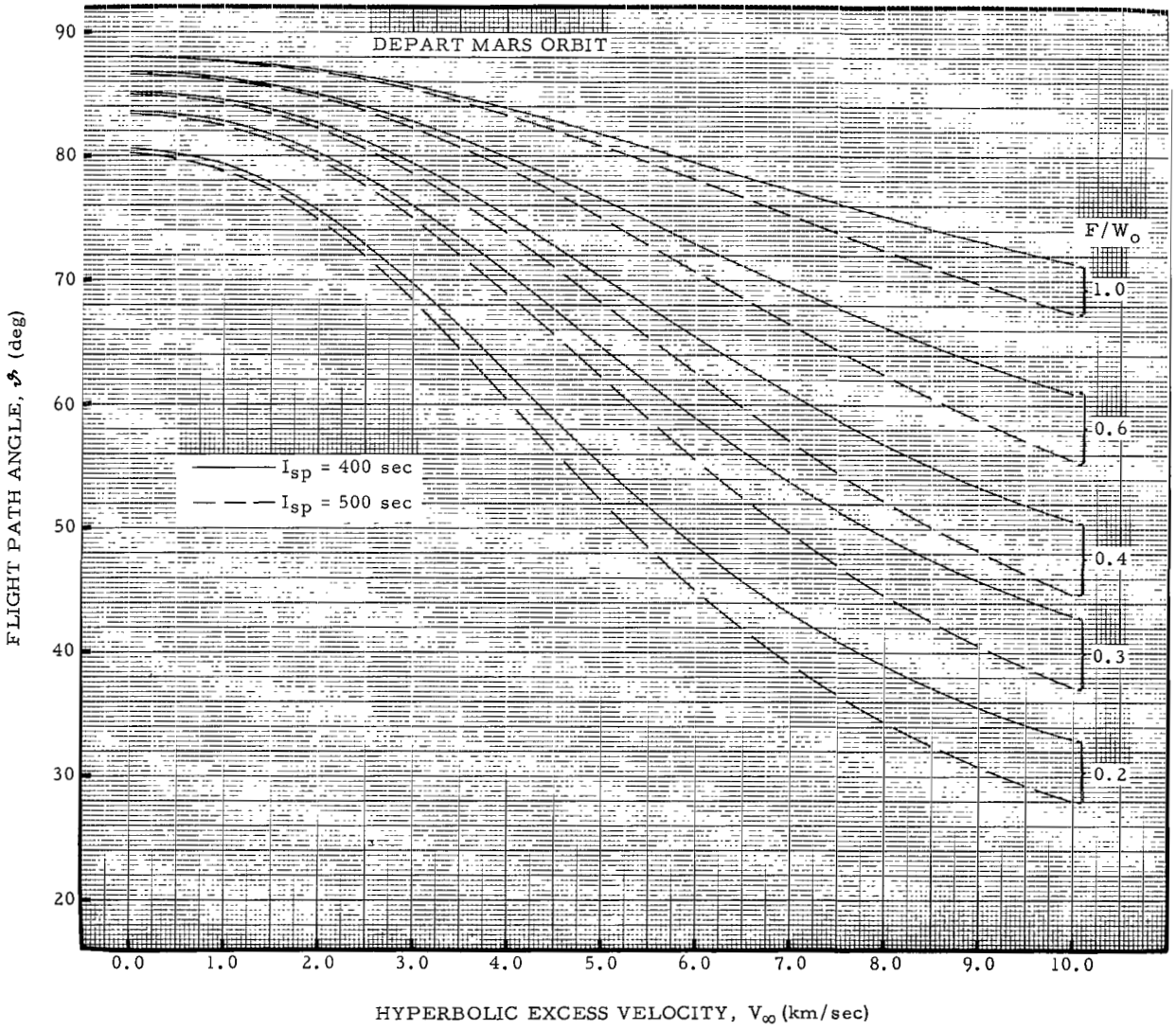


FIGURE 8. FLIGHT PATH ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

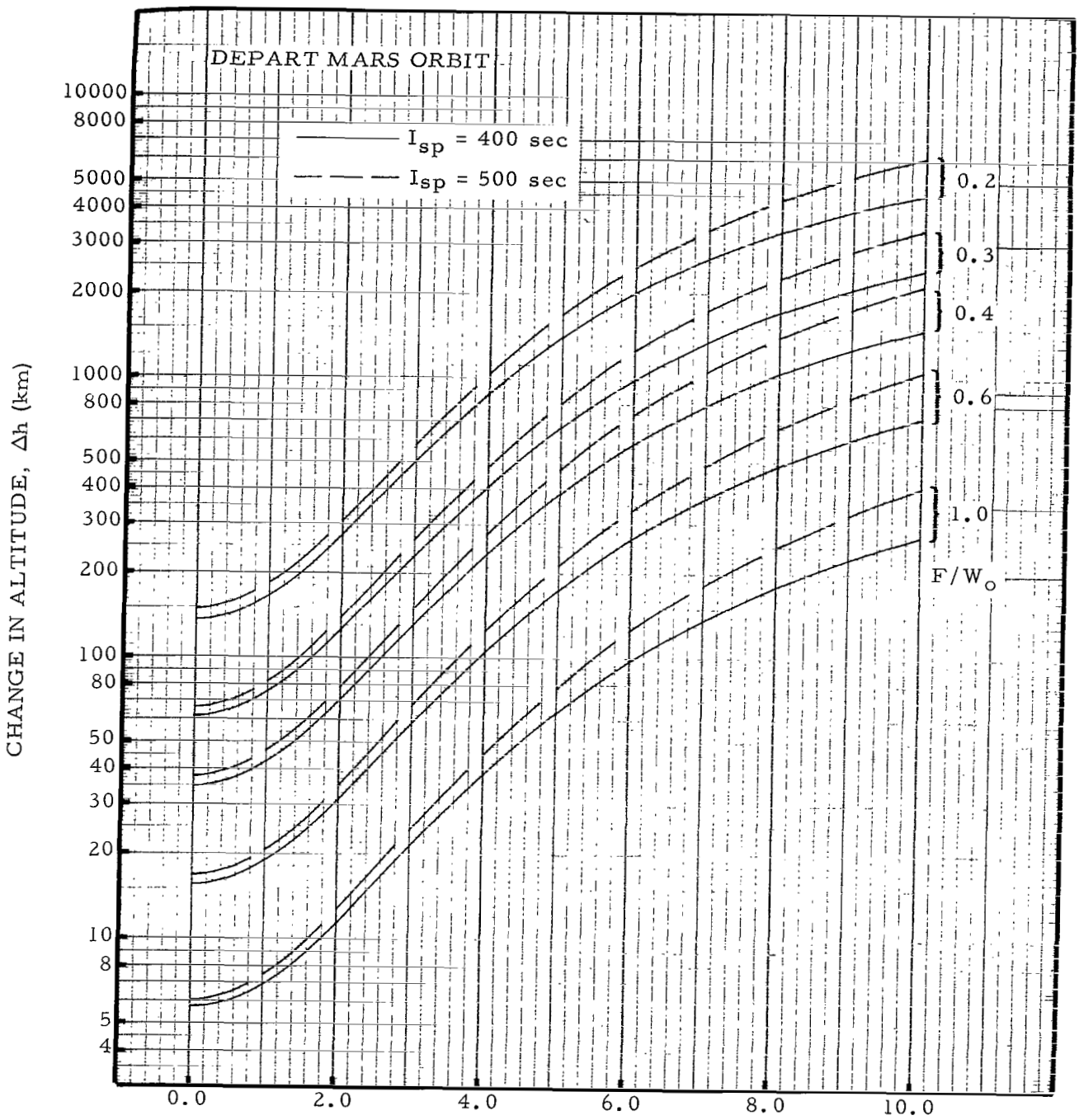


FIGURE 9. CHANGE IN ALTITUDE (km) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

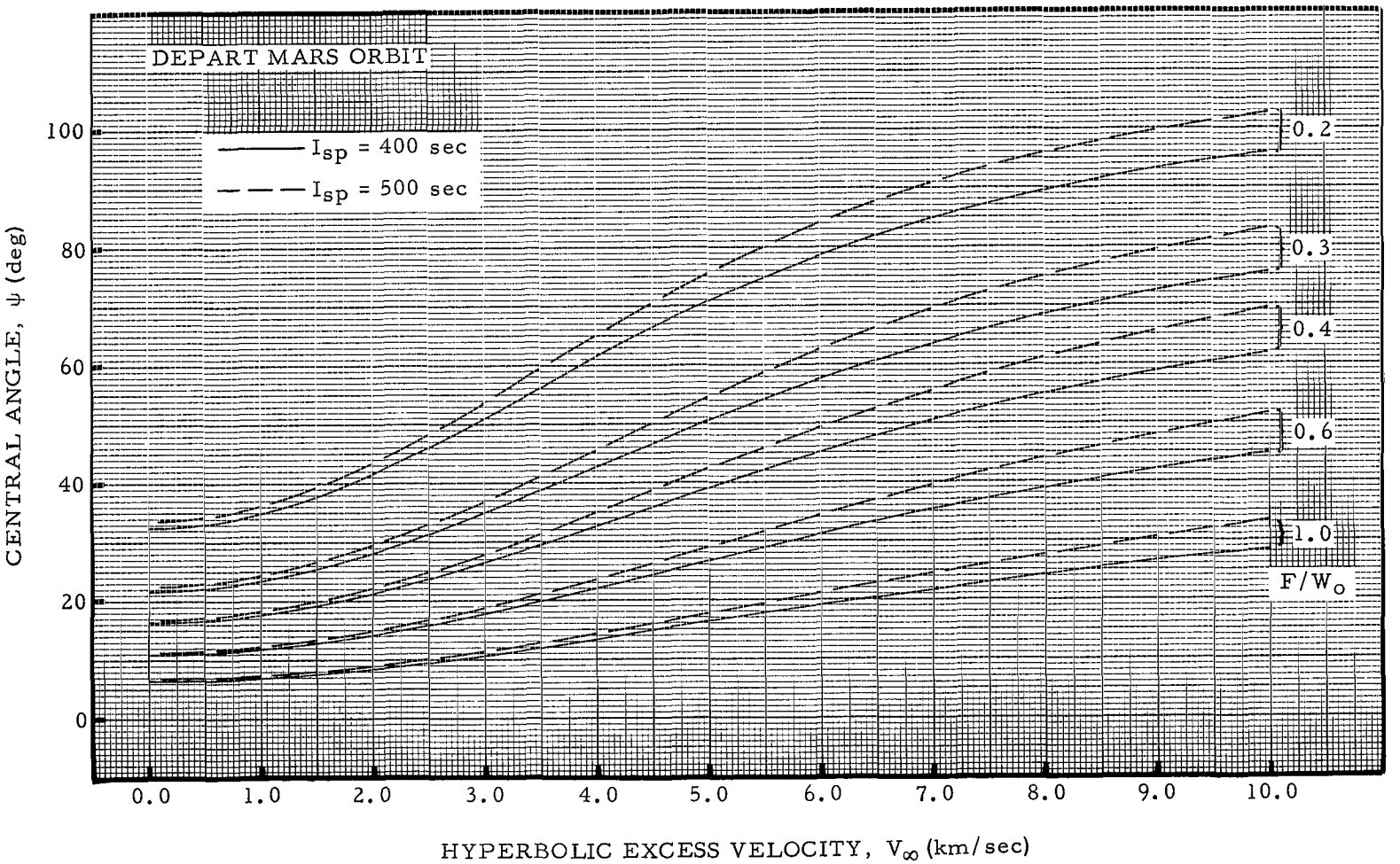


FIGURE 10. CENTRAL ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

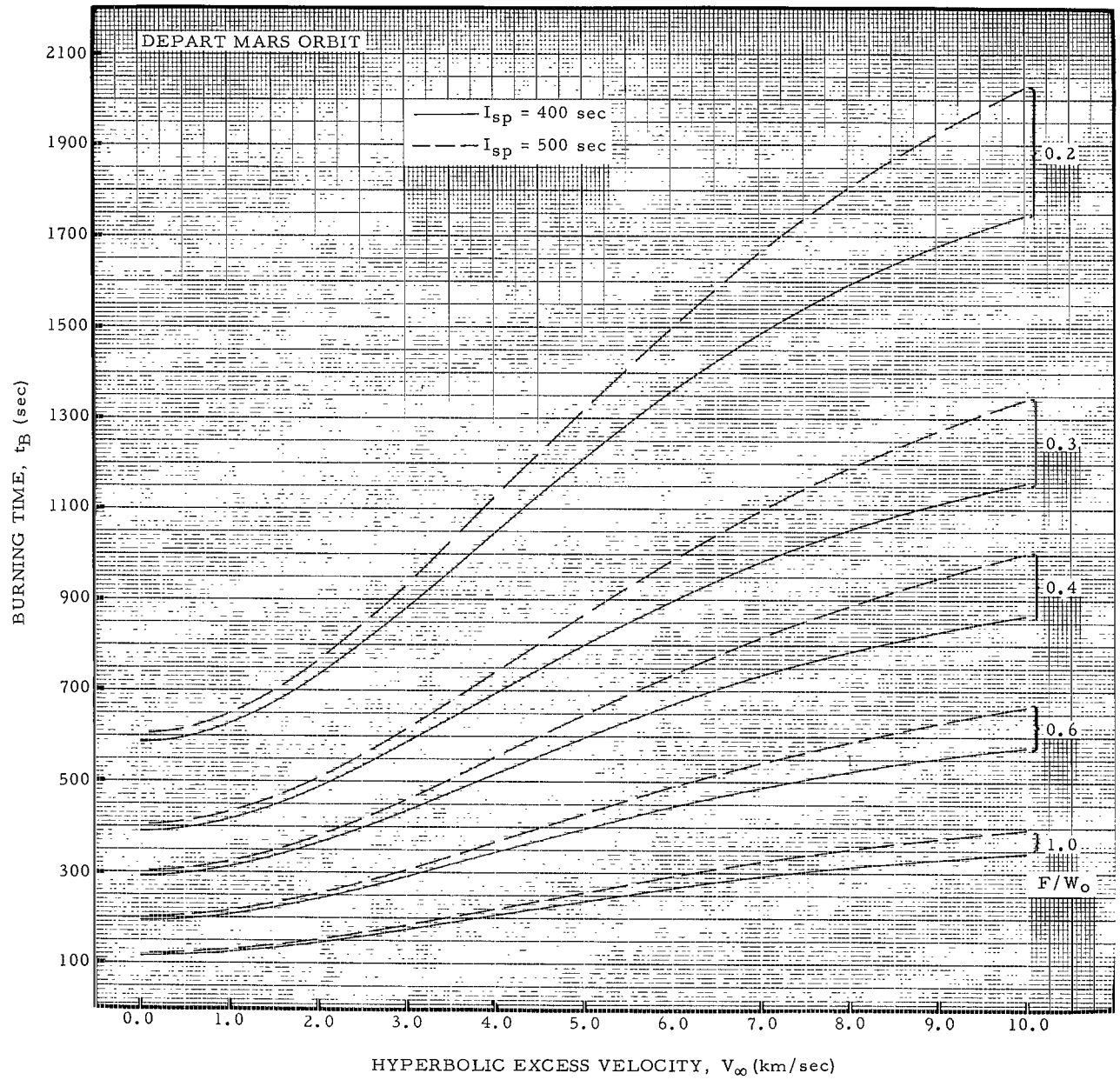


FIGURE 11. BURNING TIME (sec) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

BRAKE TO VENUS ORBIT

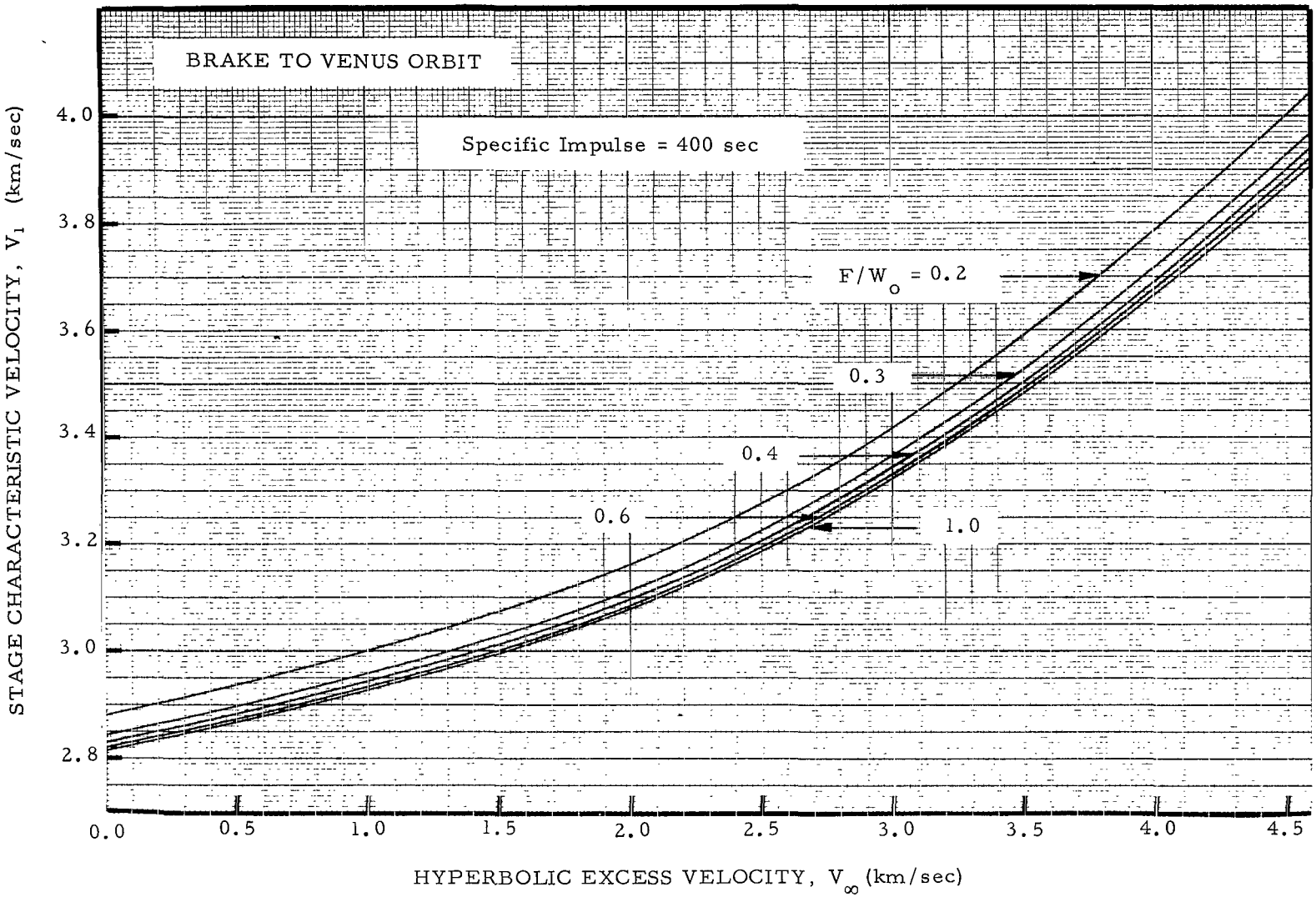


FIGURE 1a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

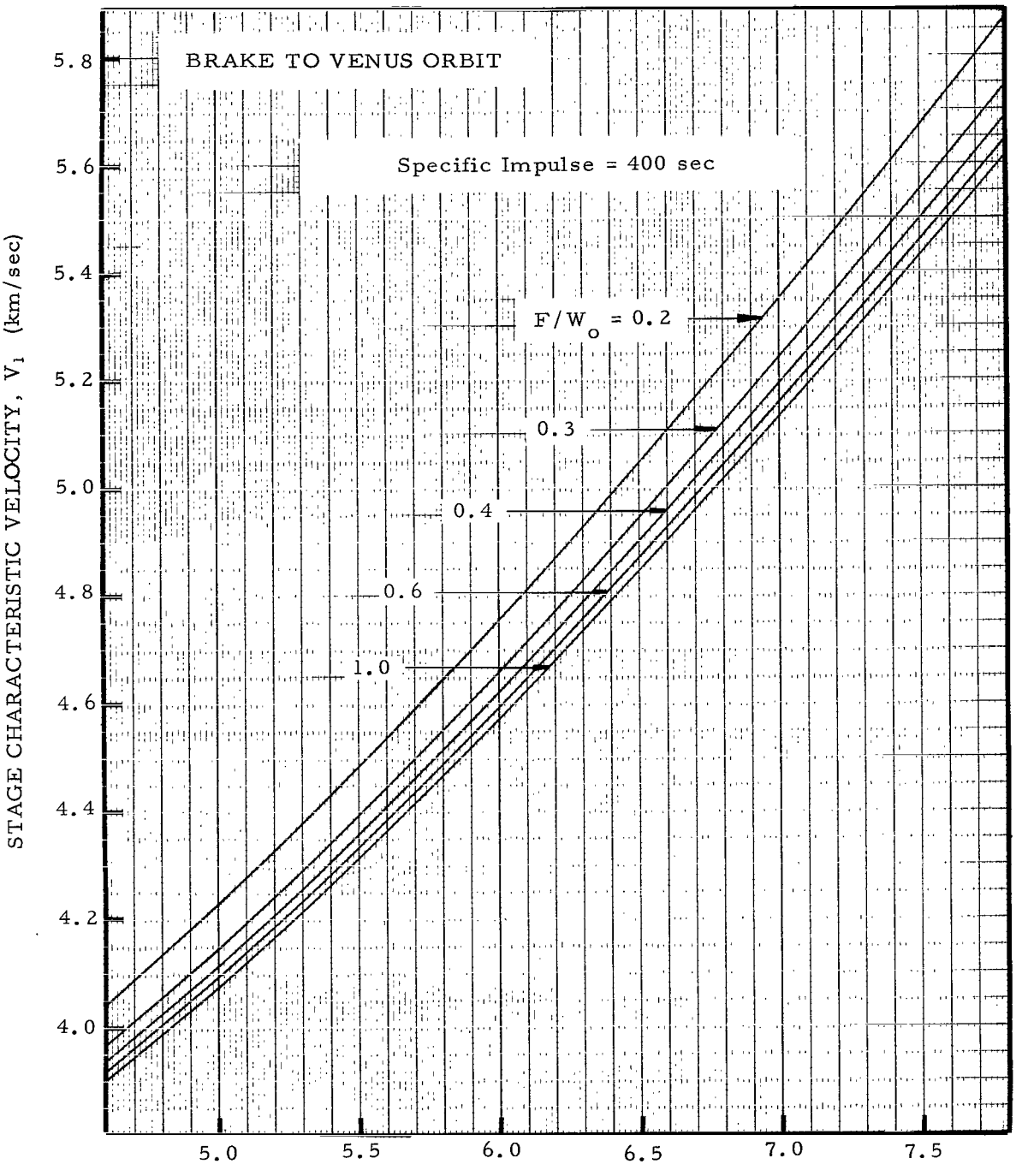


FIGURE 1b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

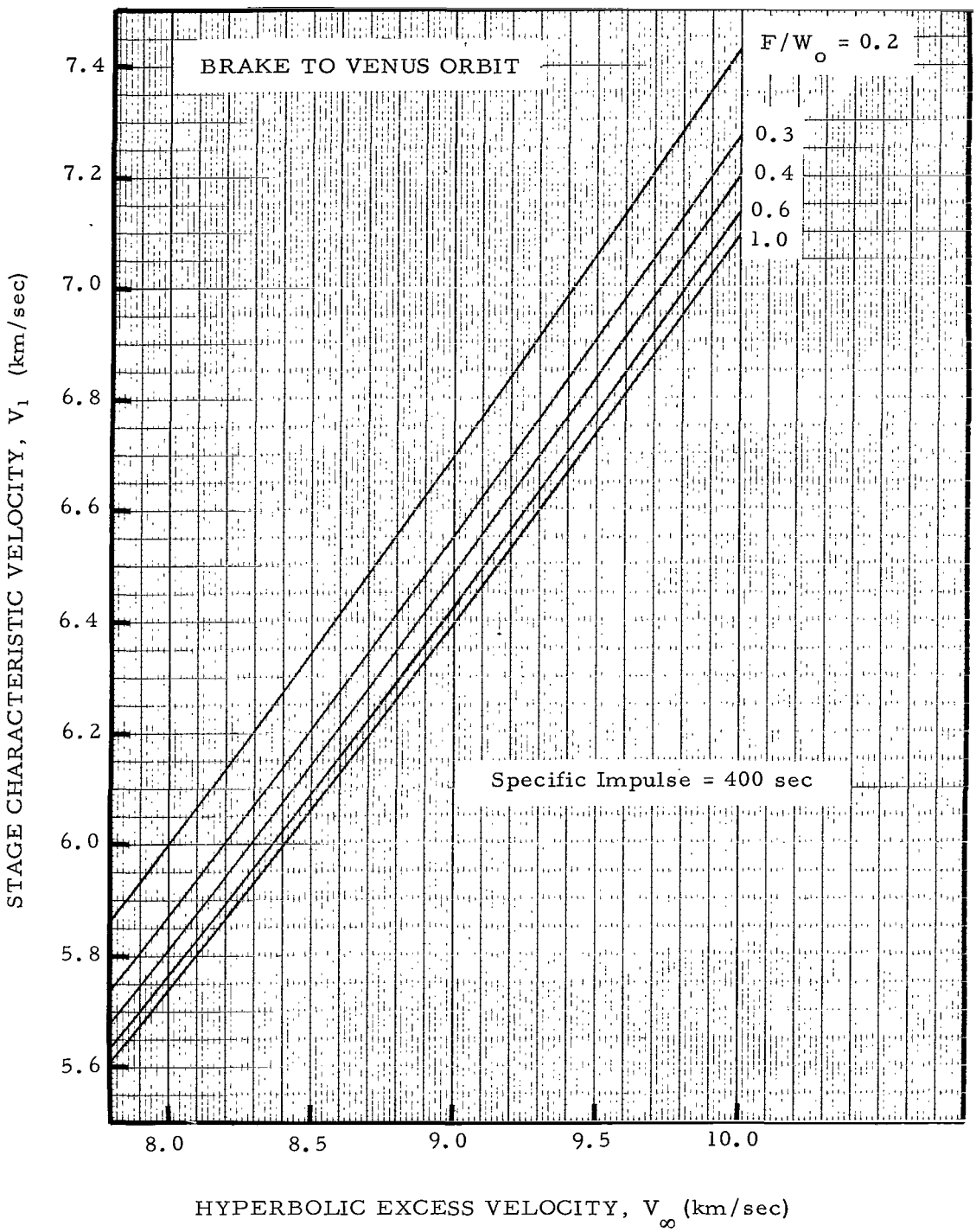


FIGURE 1c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

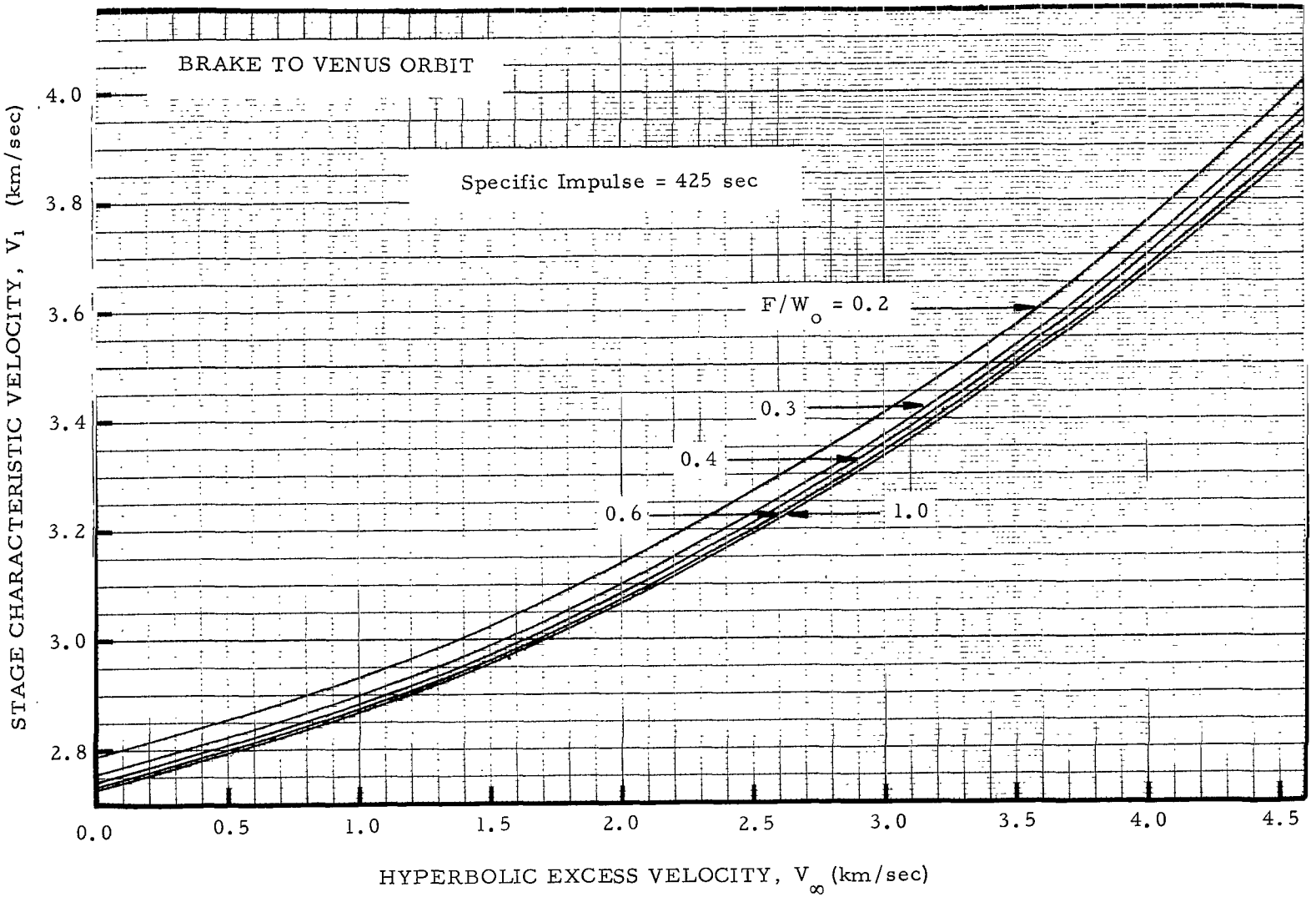
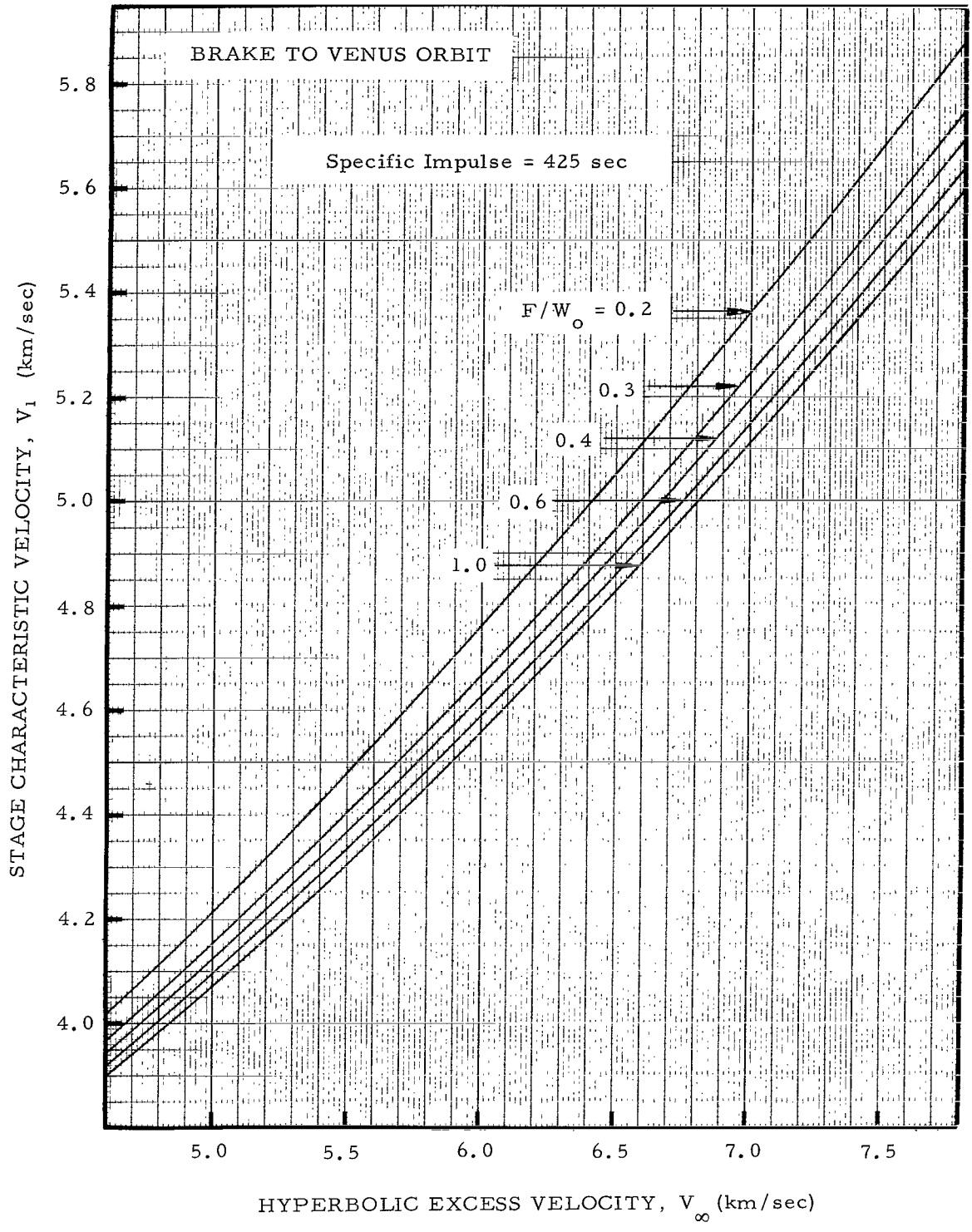


FIGURE 2a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS



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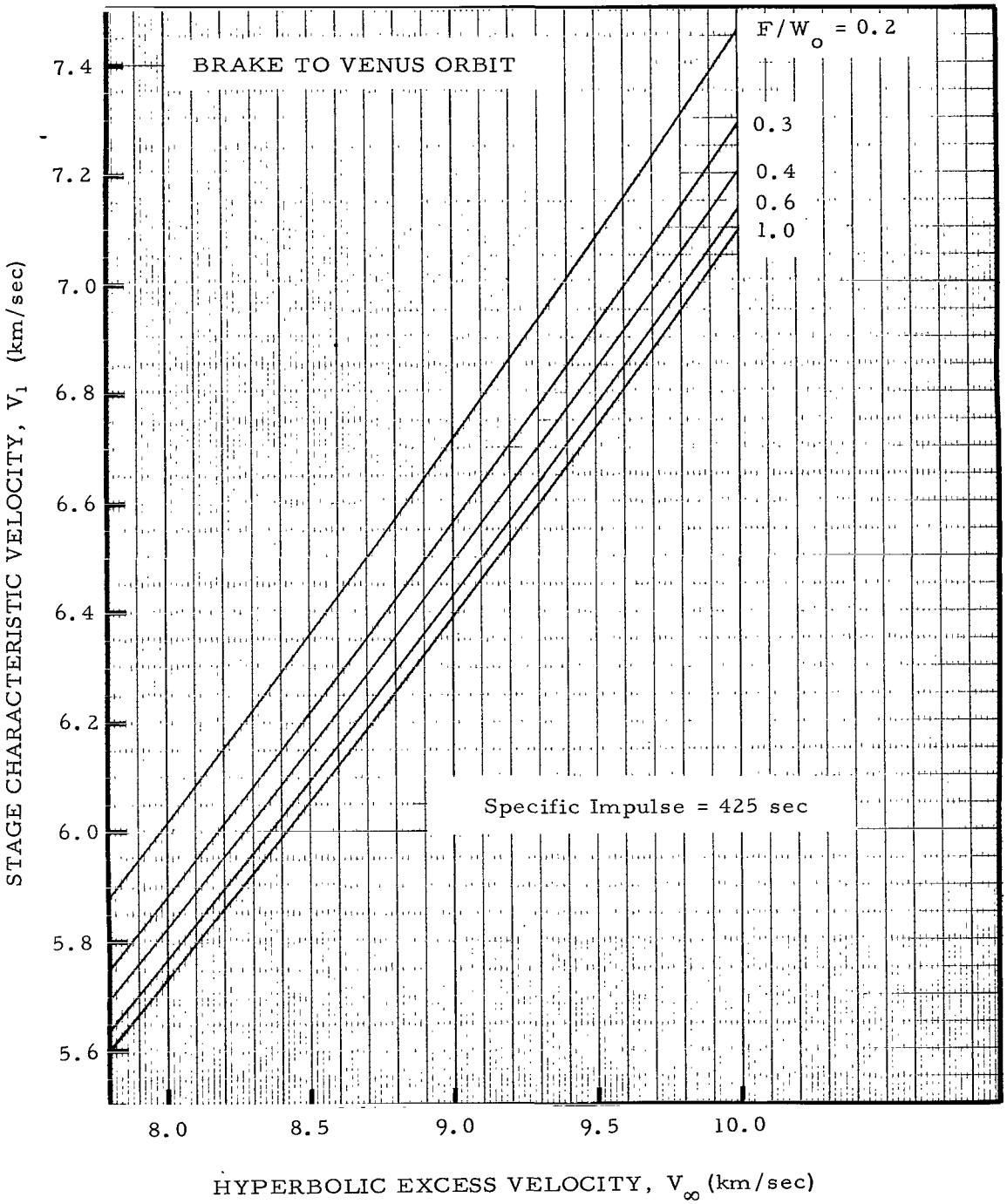
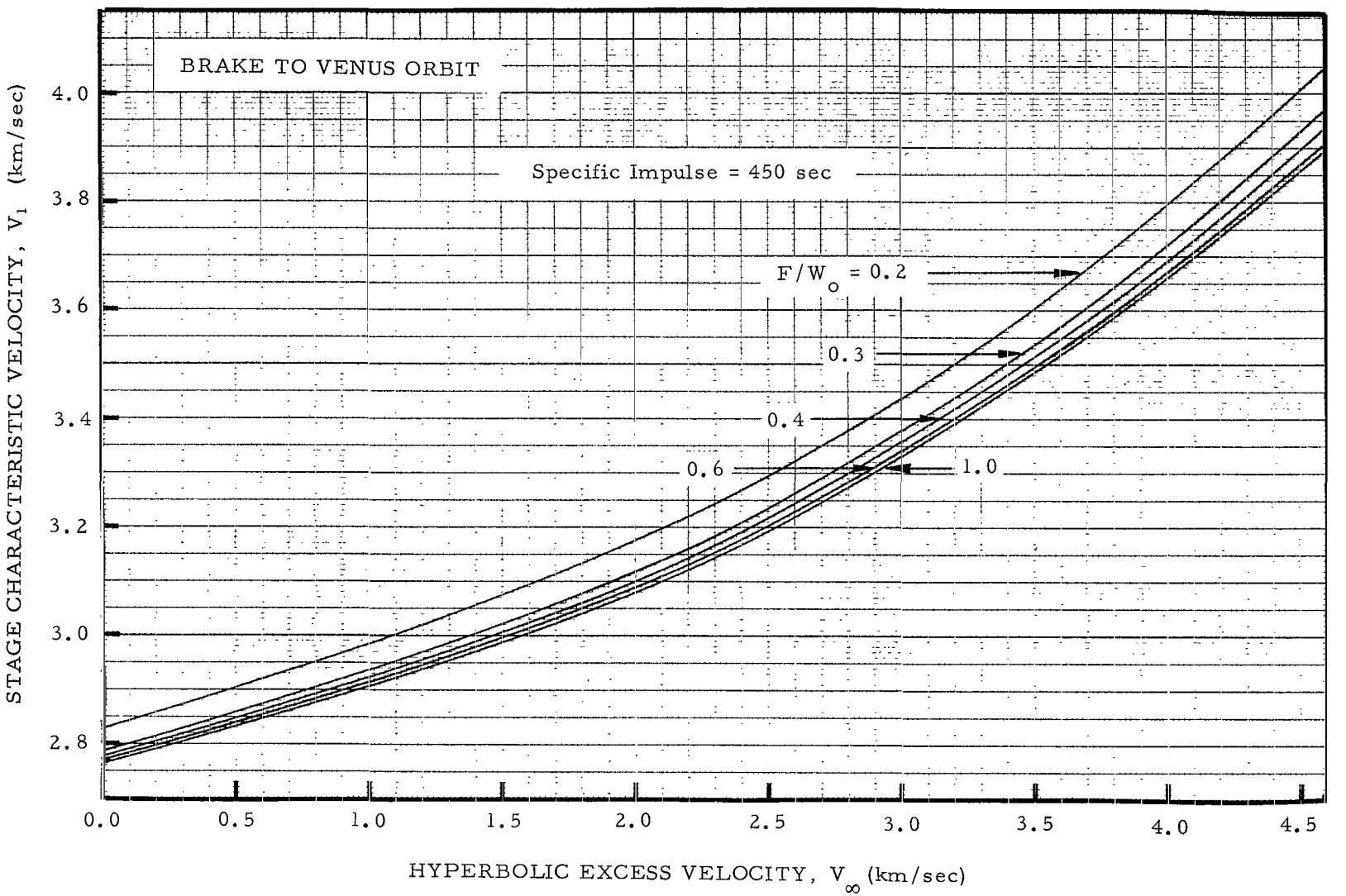


FIGURE 2c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS



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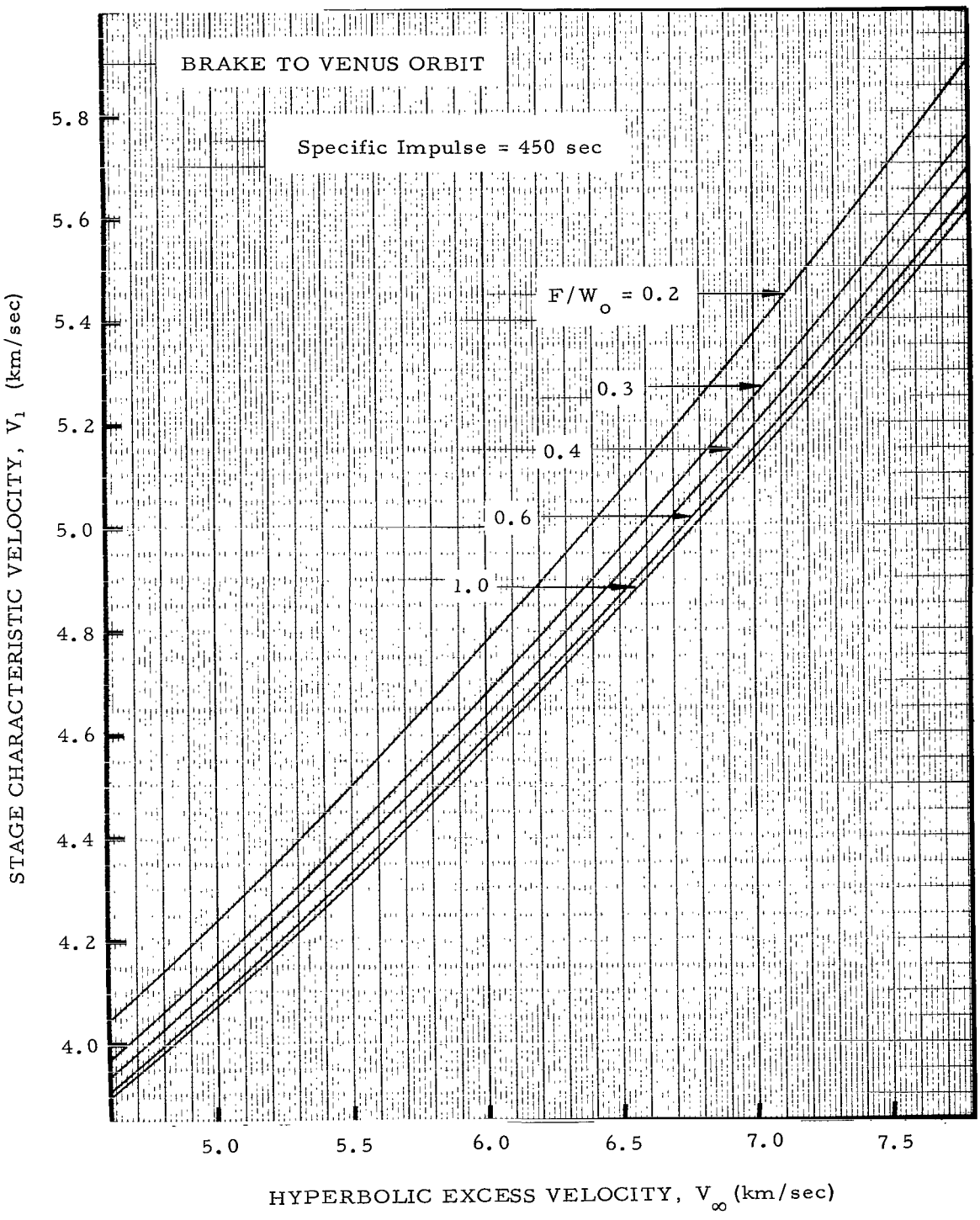


FIGURE 3b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

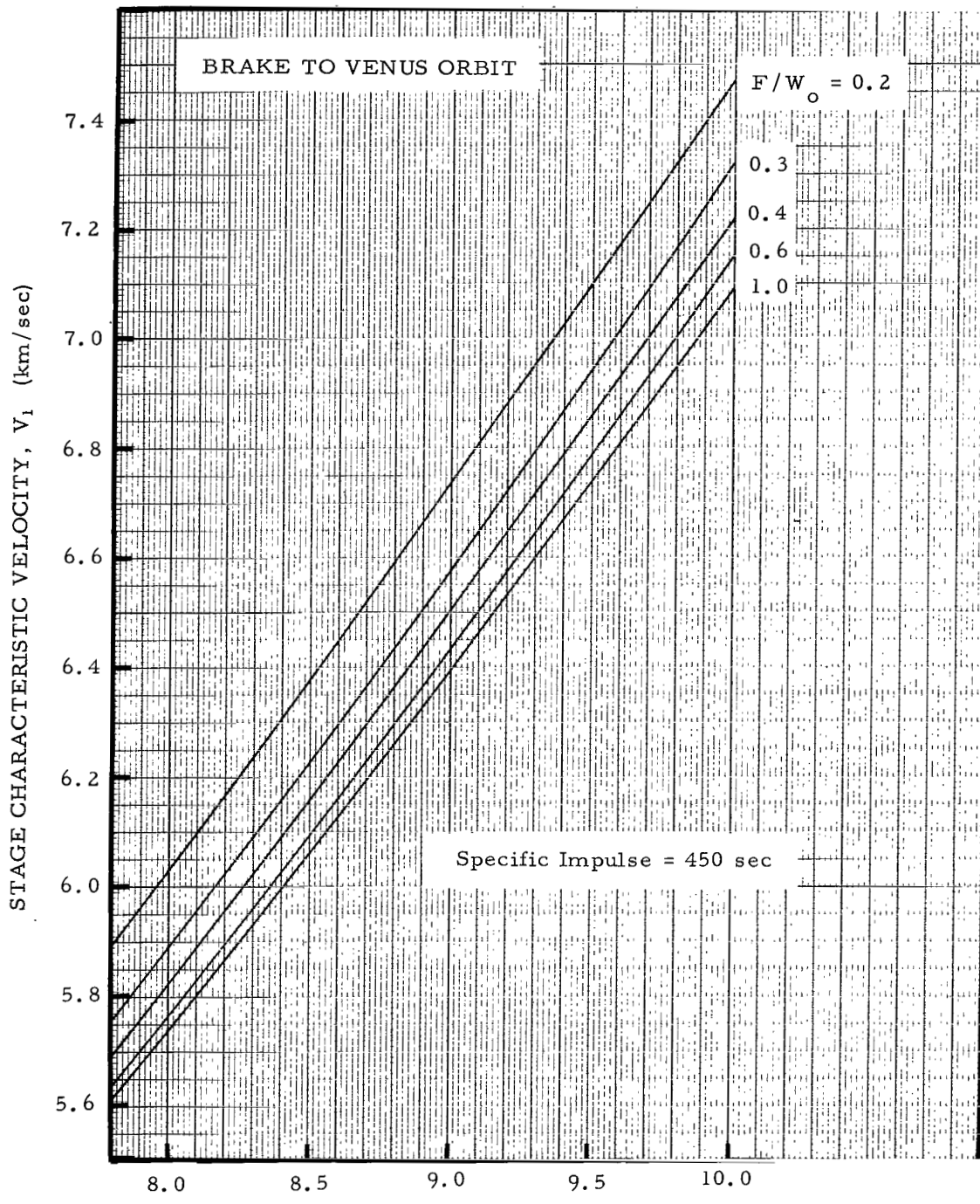


FIGURE 3c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

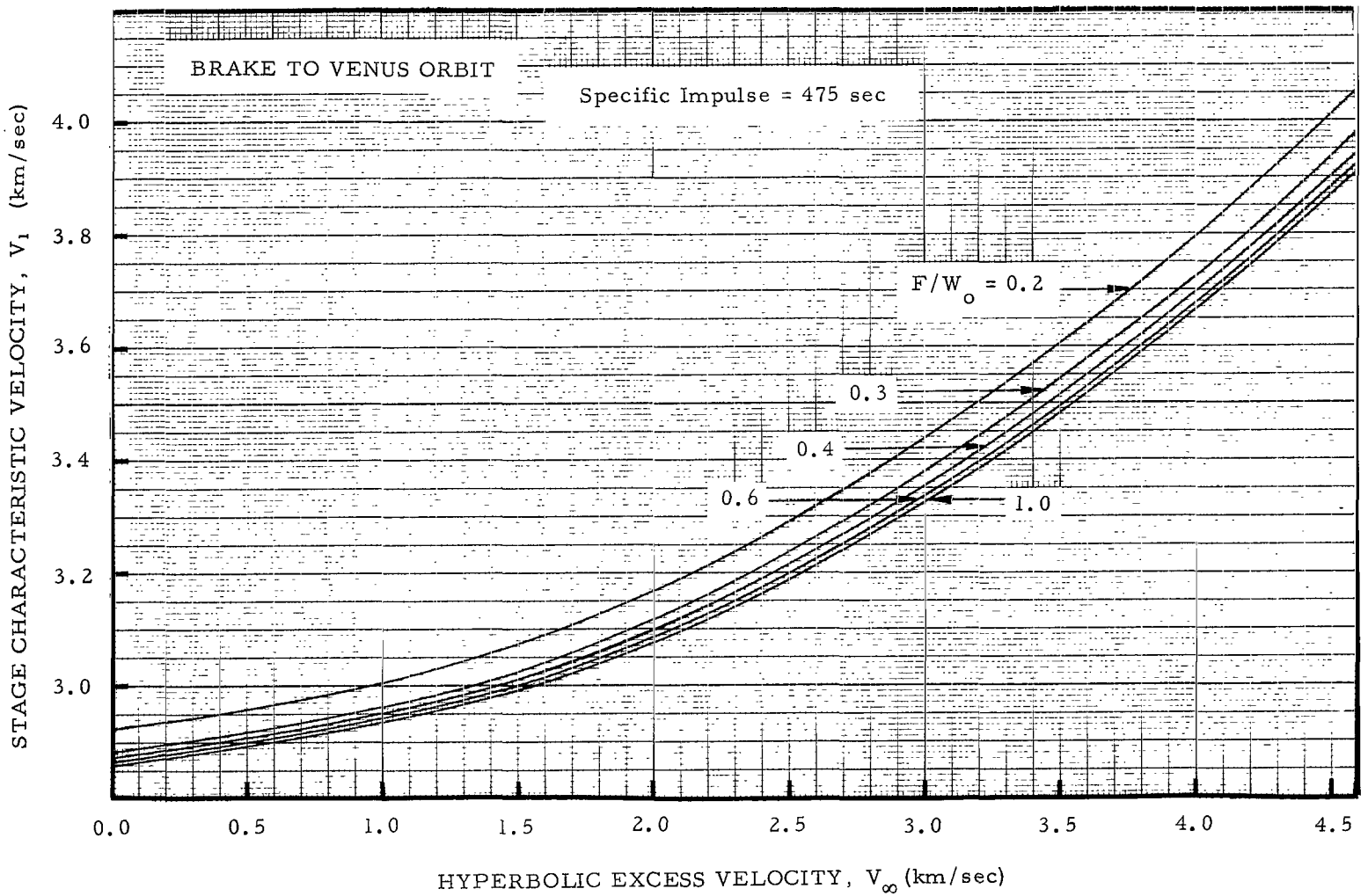


FIGURE 4a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

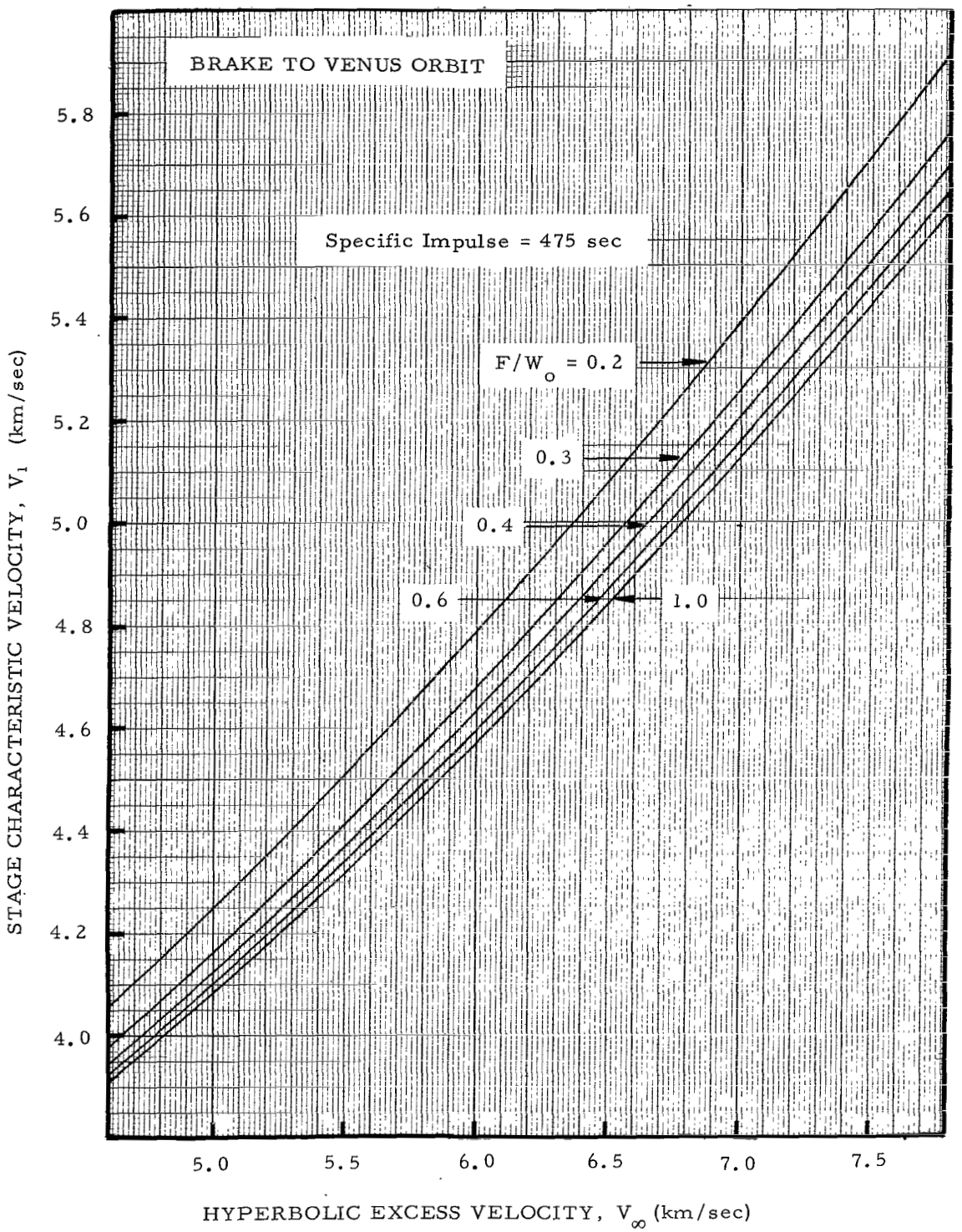
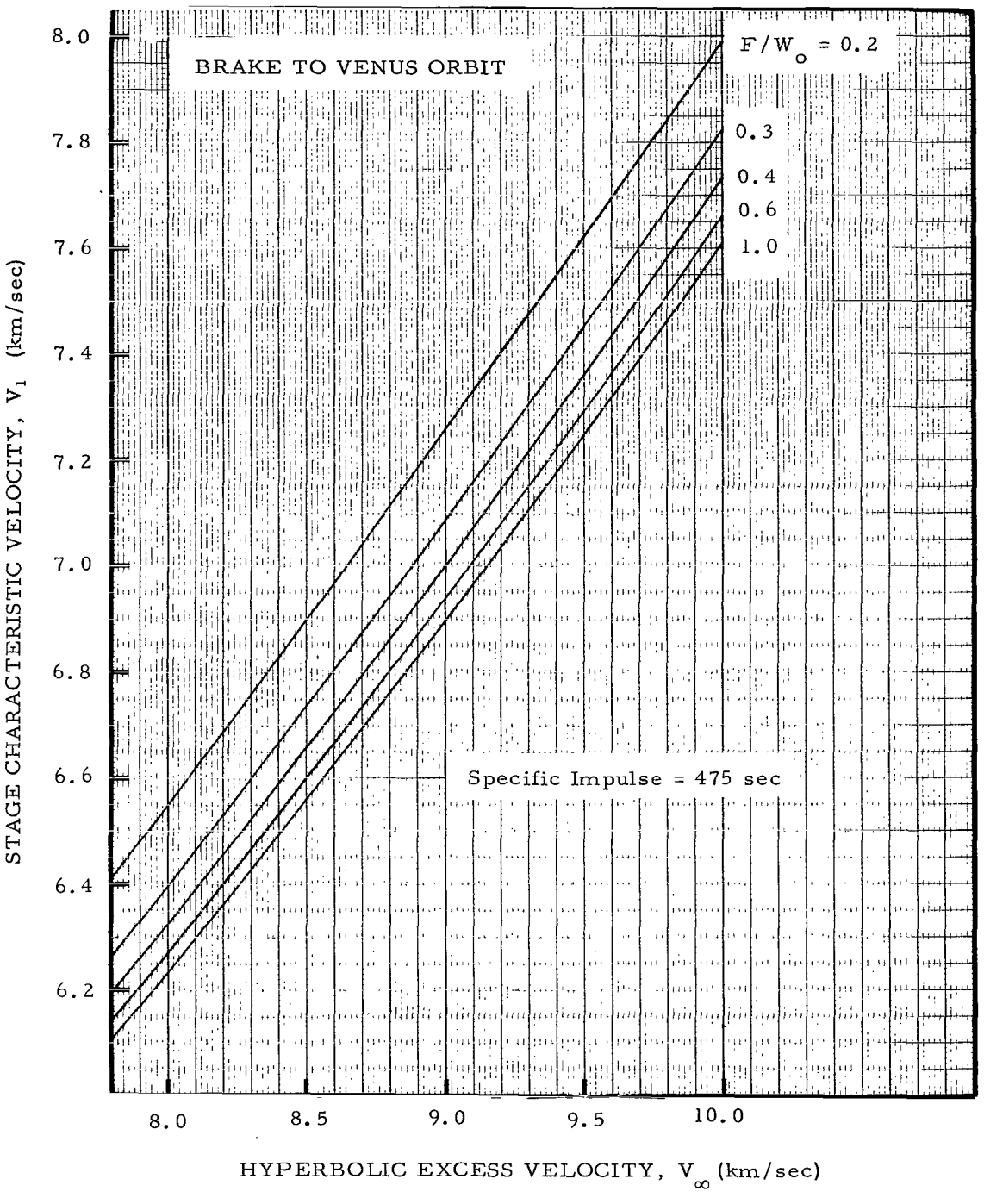


FIGURE 4b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS



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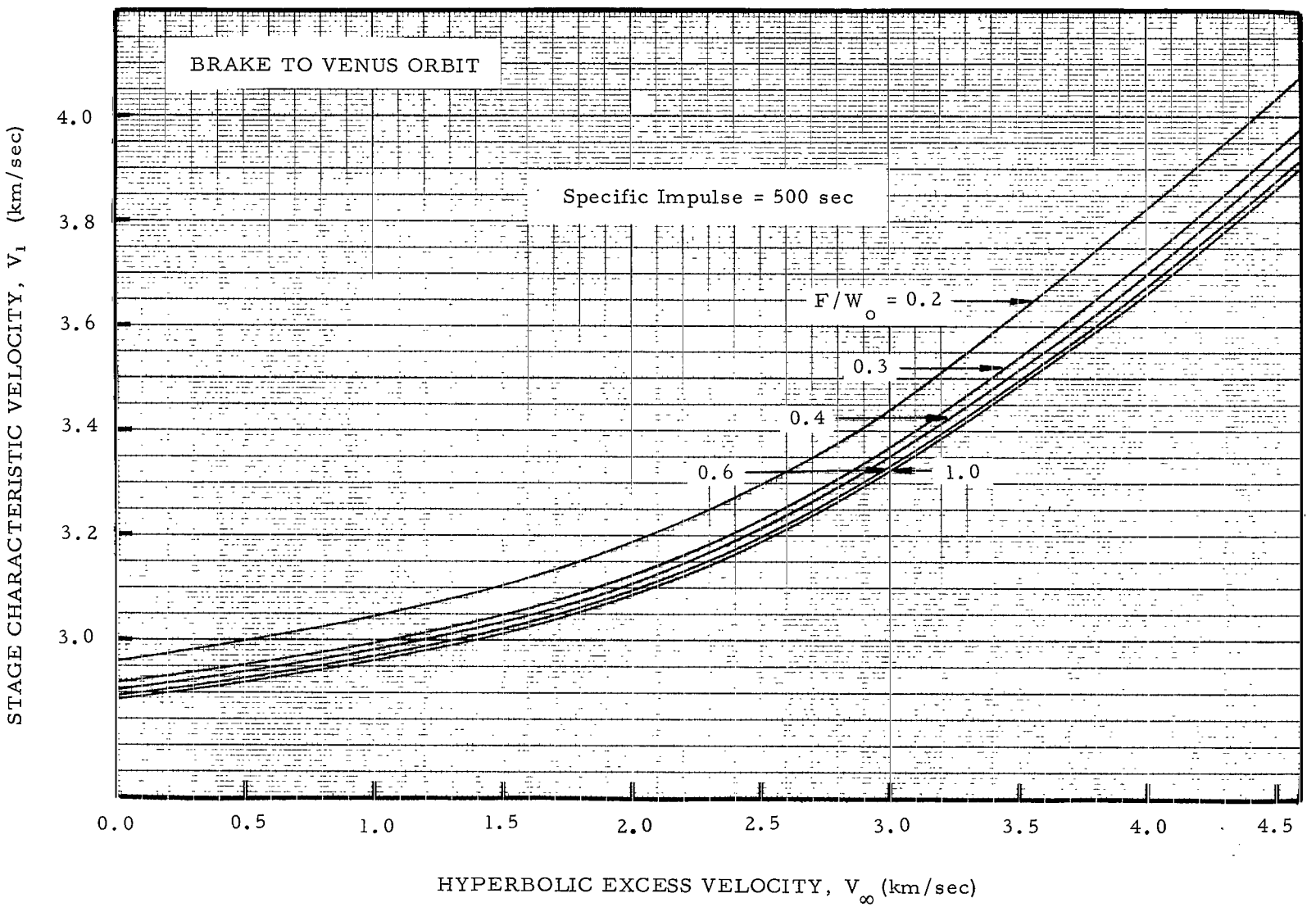


FIGURE 5a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

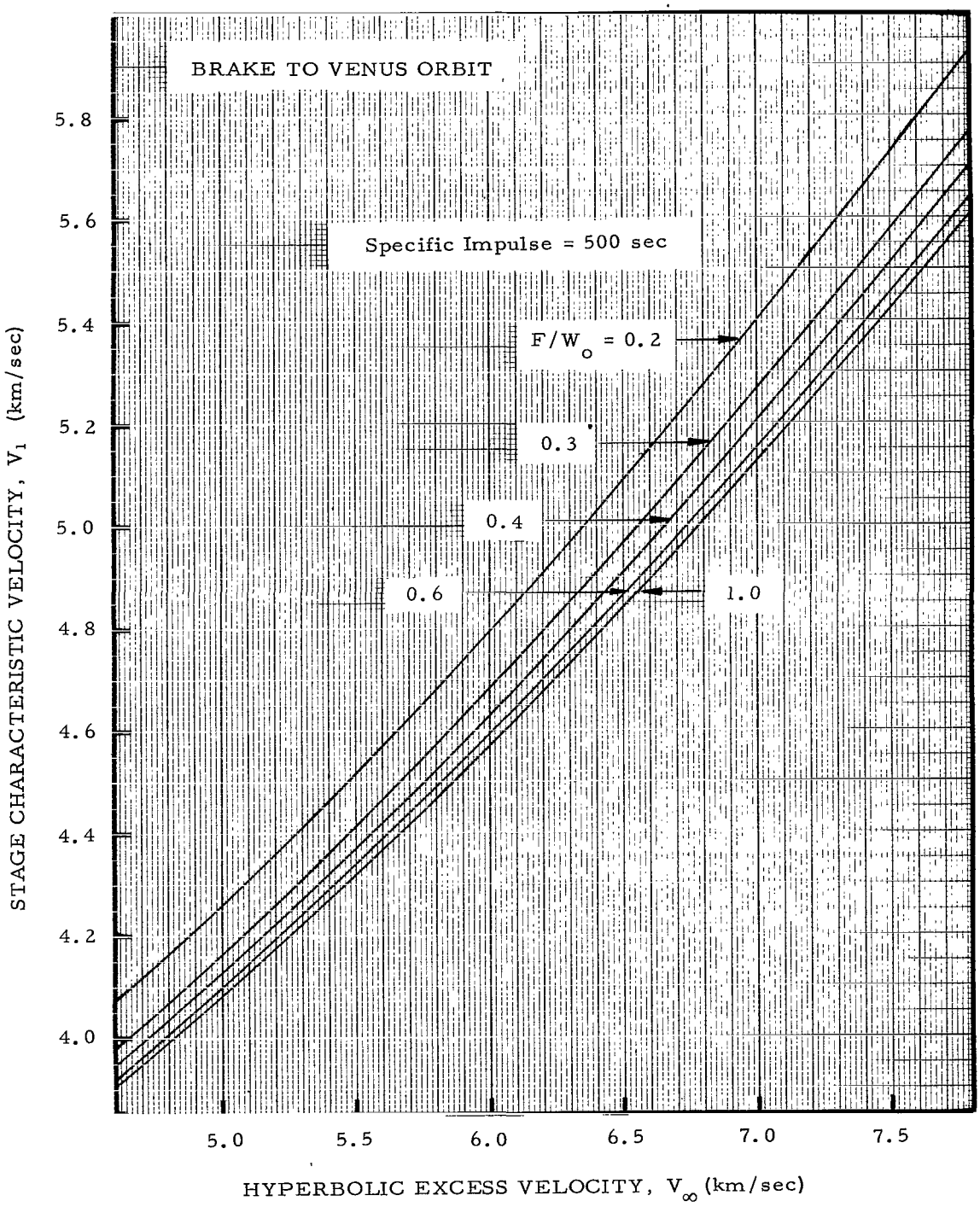
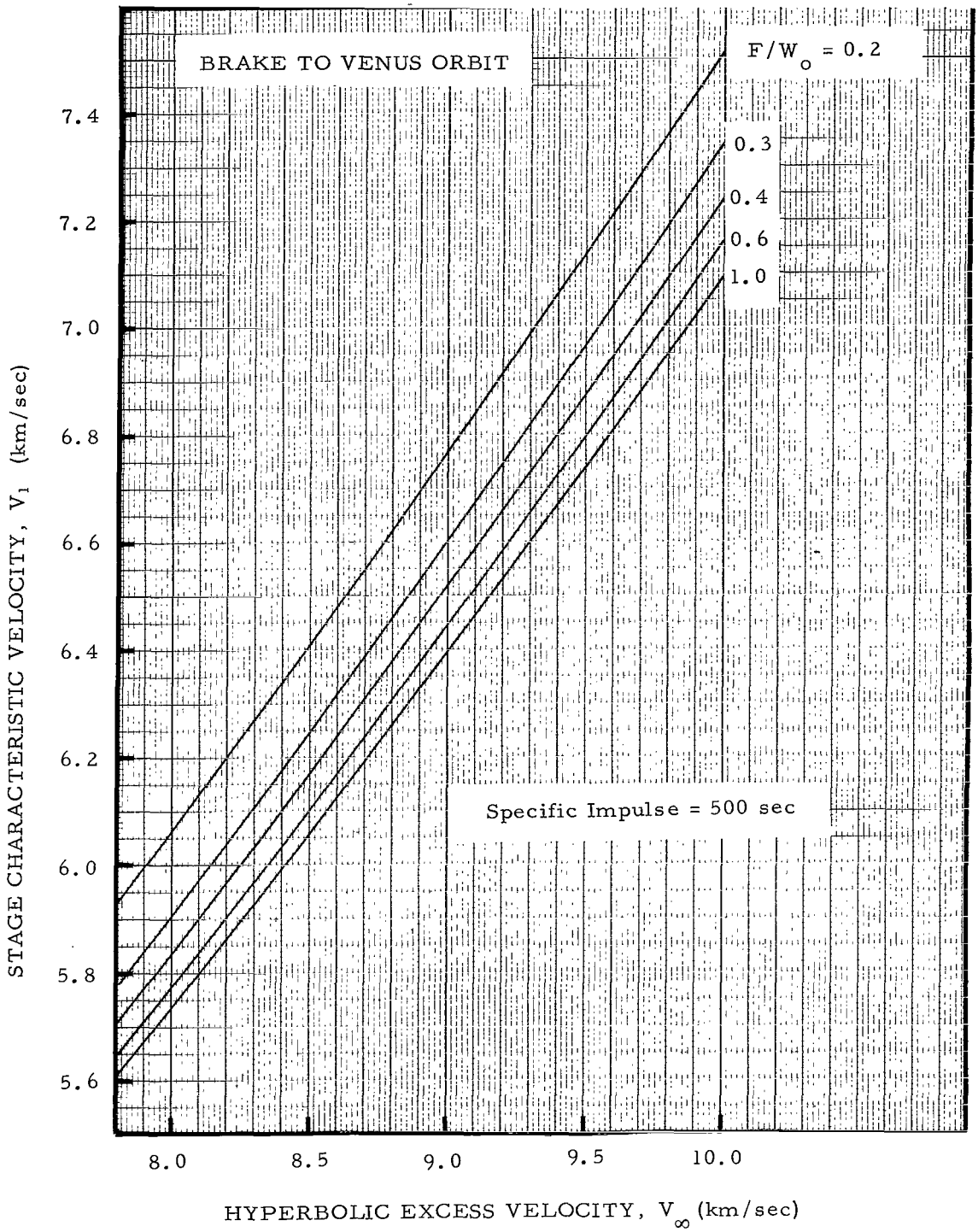


FIGURE 5b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS



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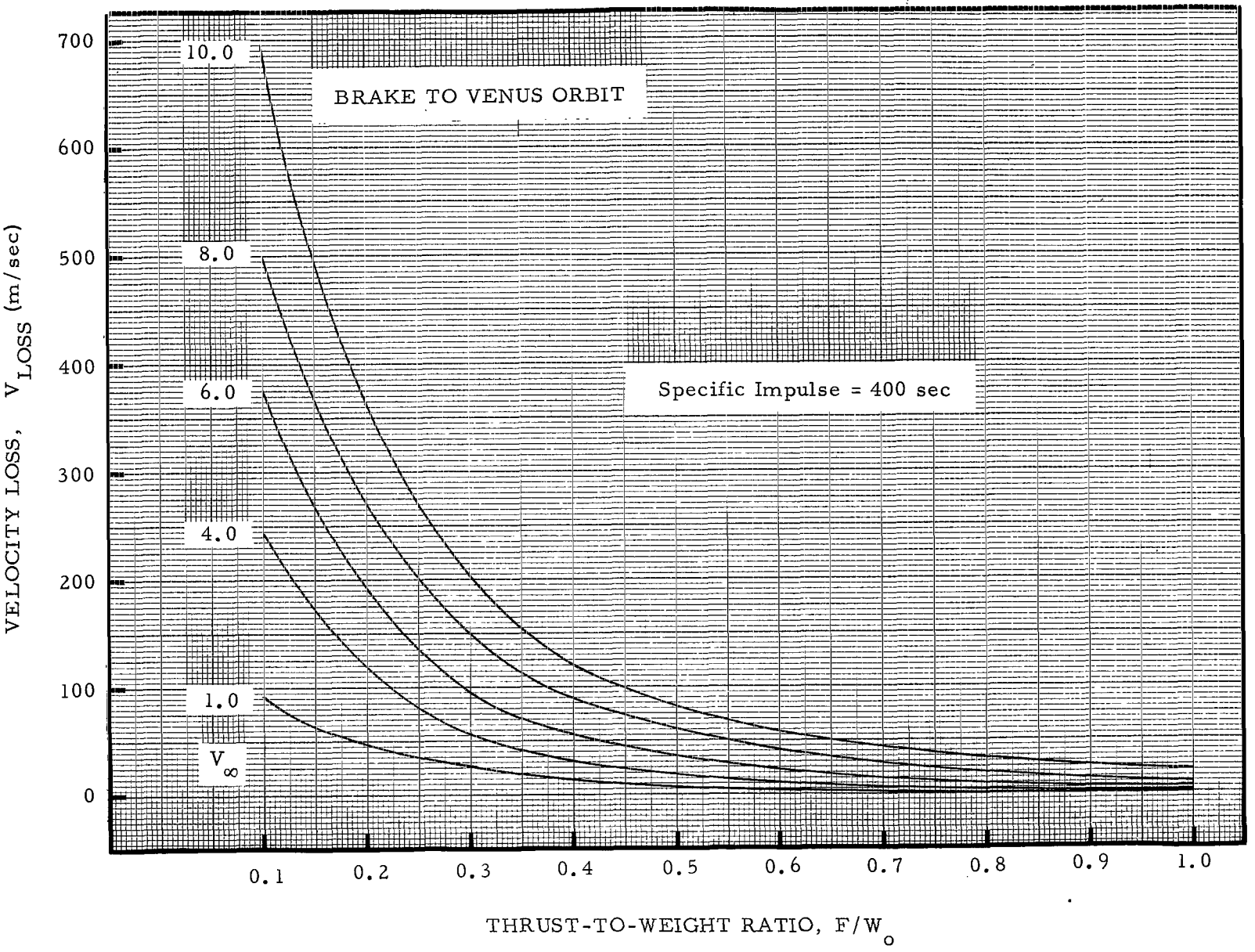
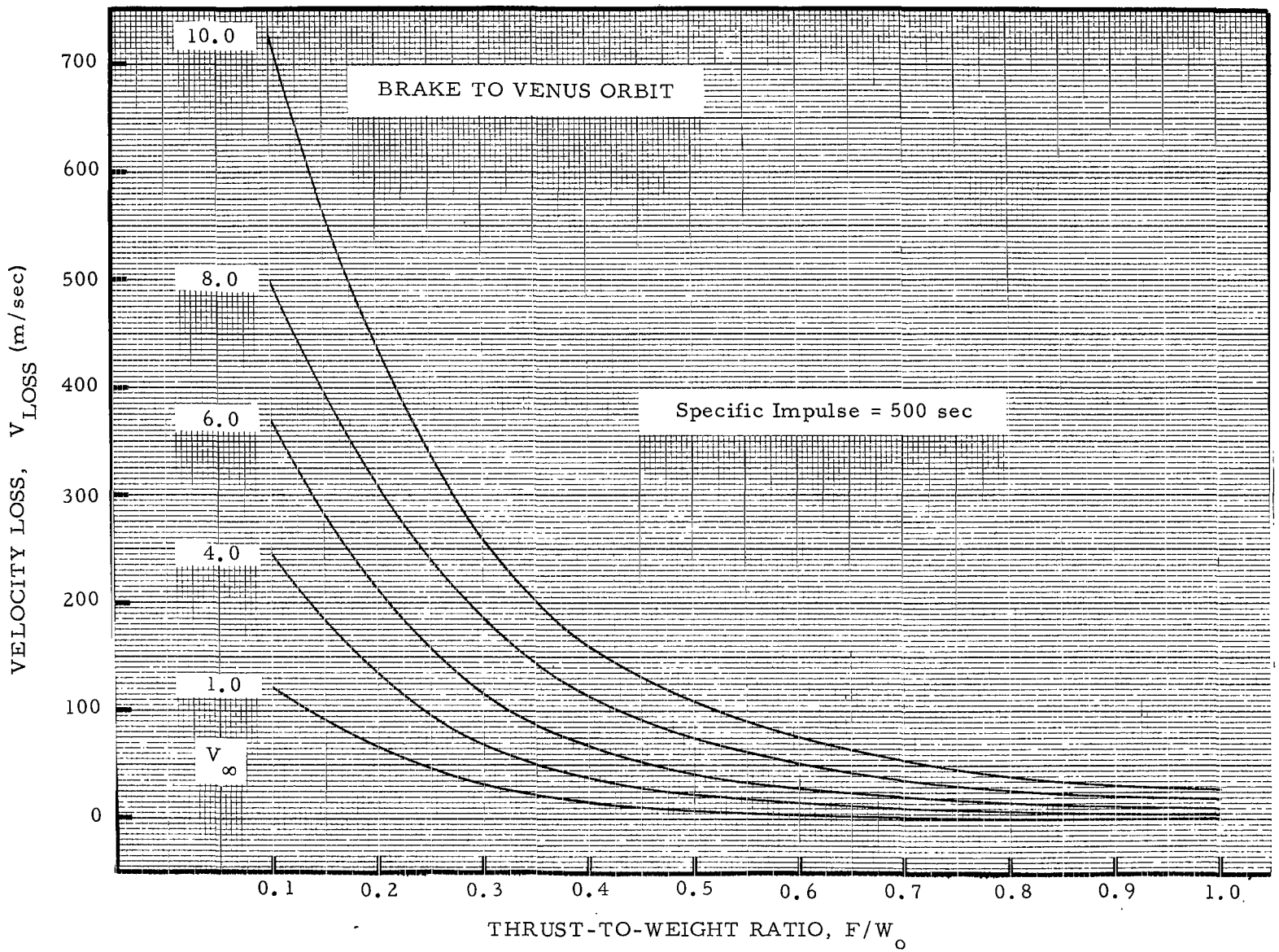


FIGURE 6. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER



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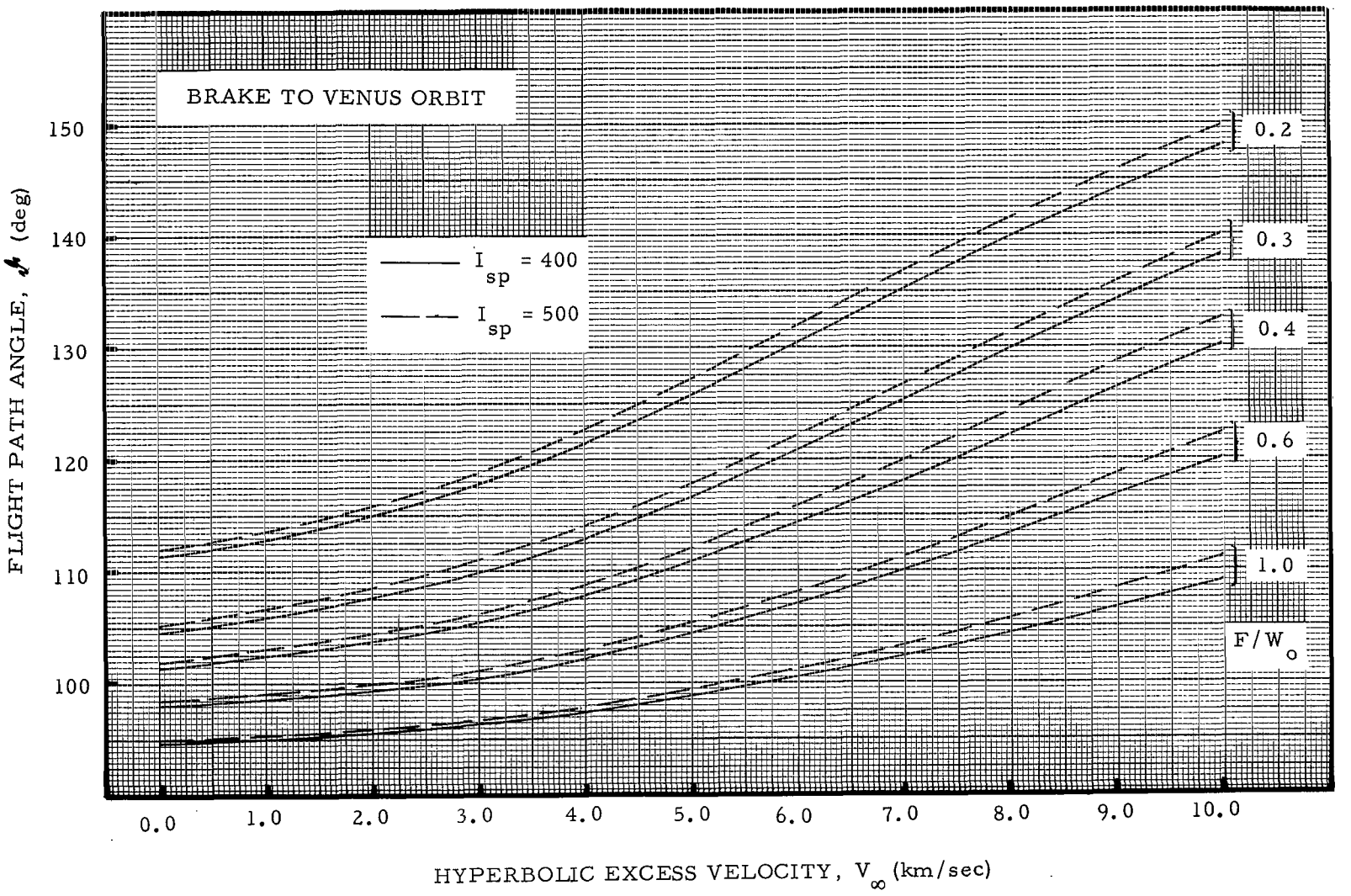


FIGURE 8. FLIGHT PATH ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

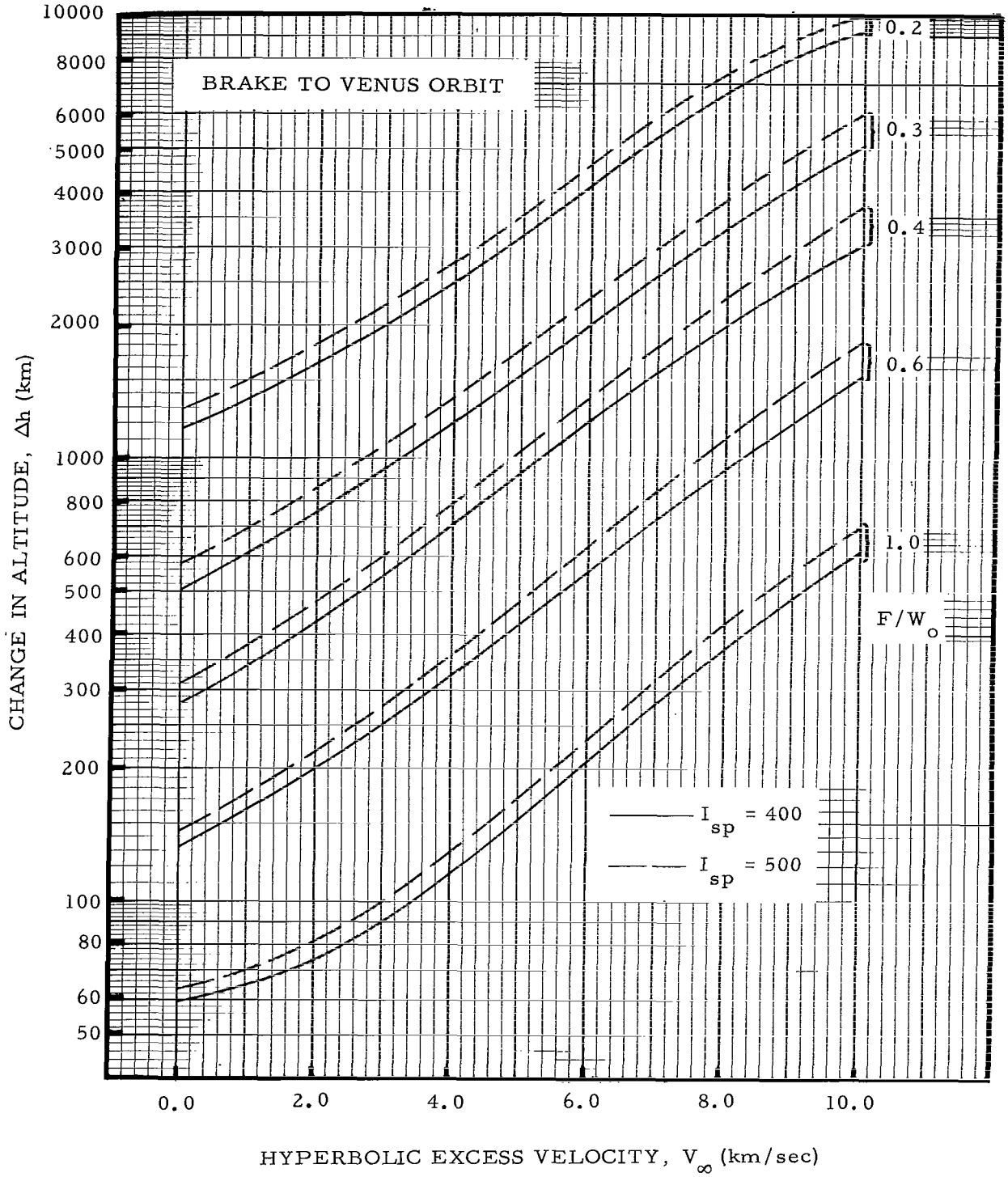


FIGURE 9. CHANGE IN ALTITUDE (km) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

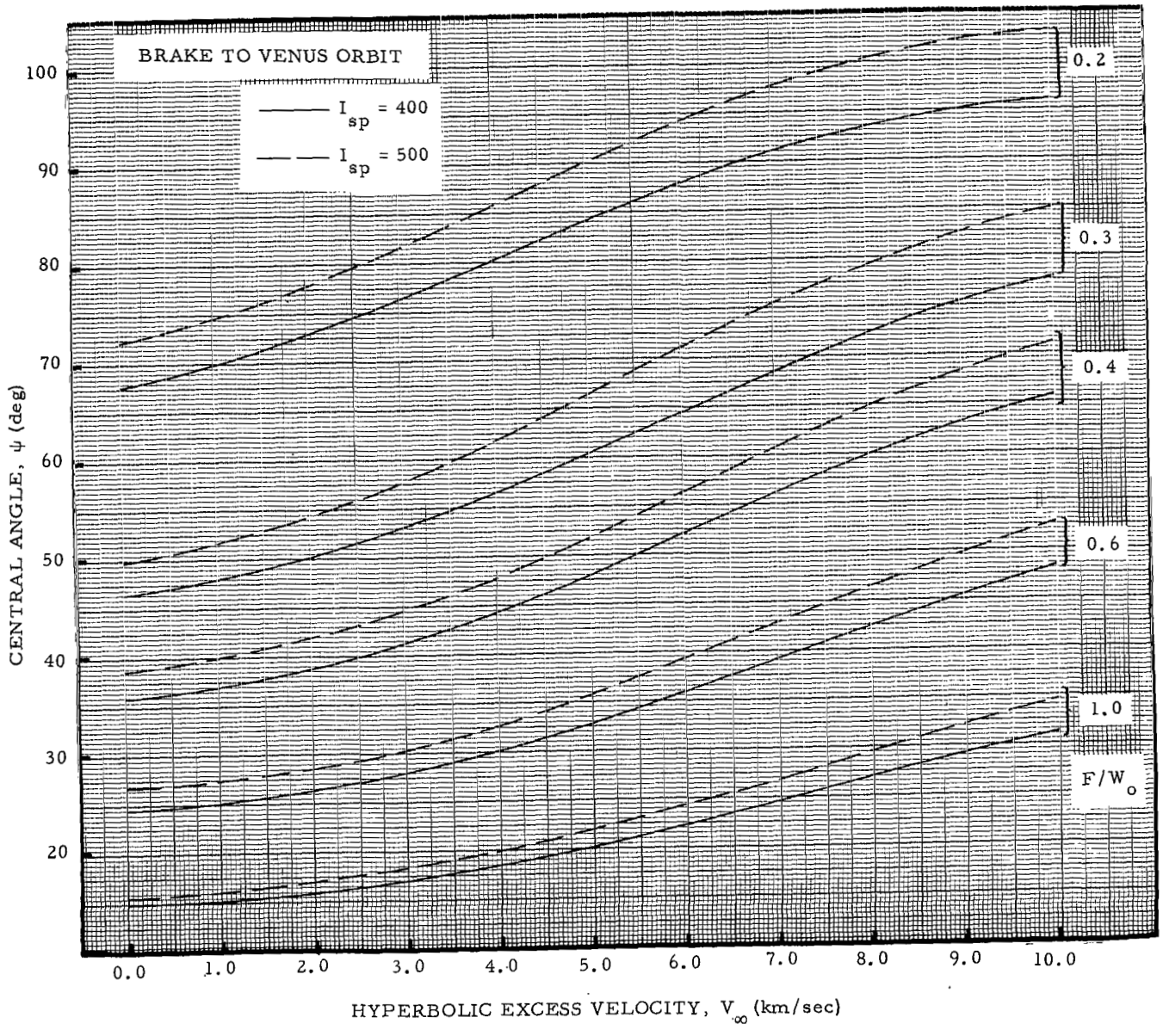


FIGURE 10. CENTRAL ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

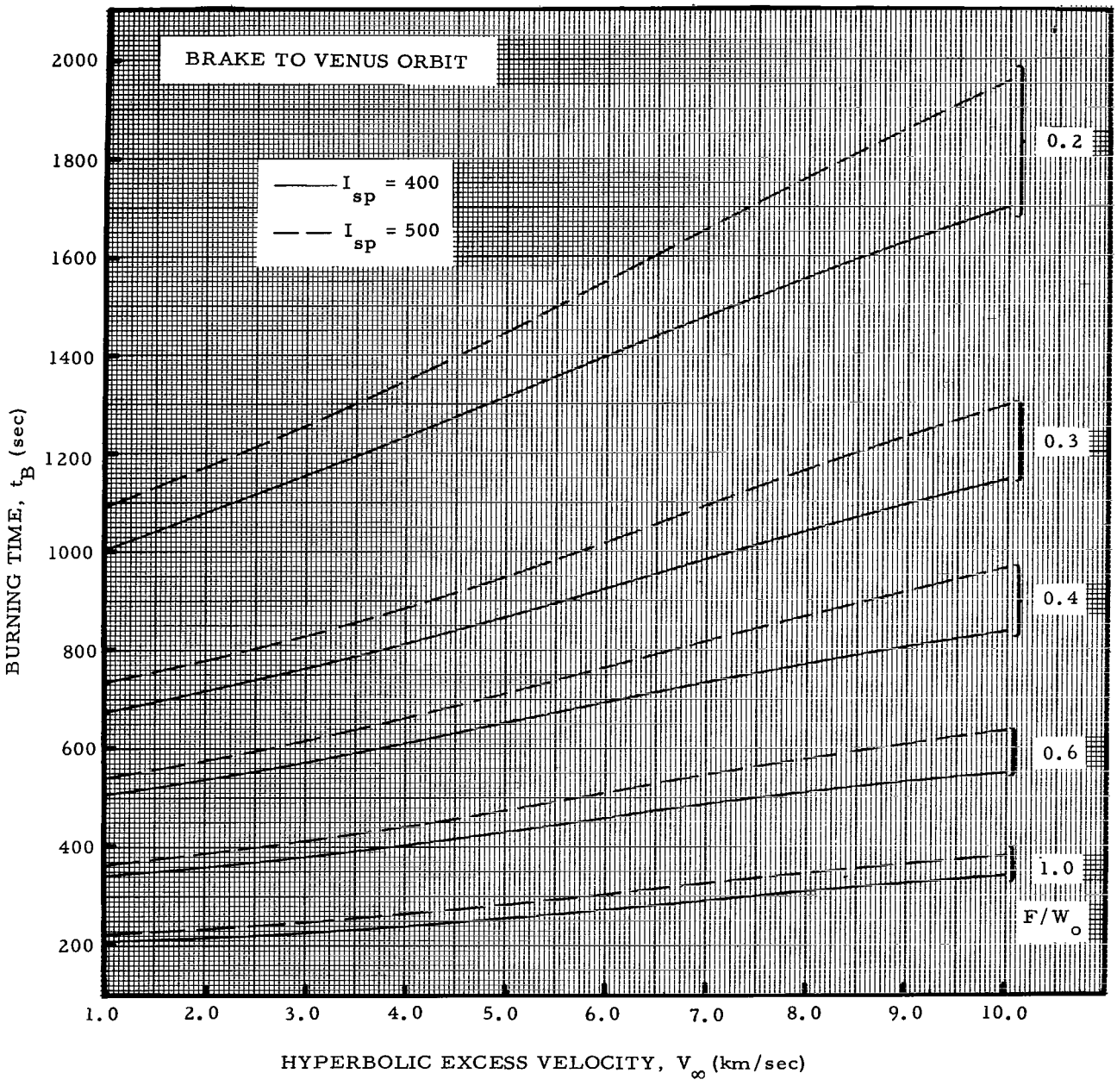


FIGURE 11. BURNING TIME (sec) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

BRAKE TO MARS ORBIT

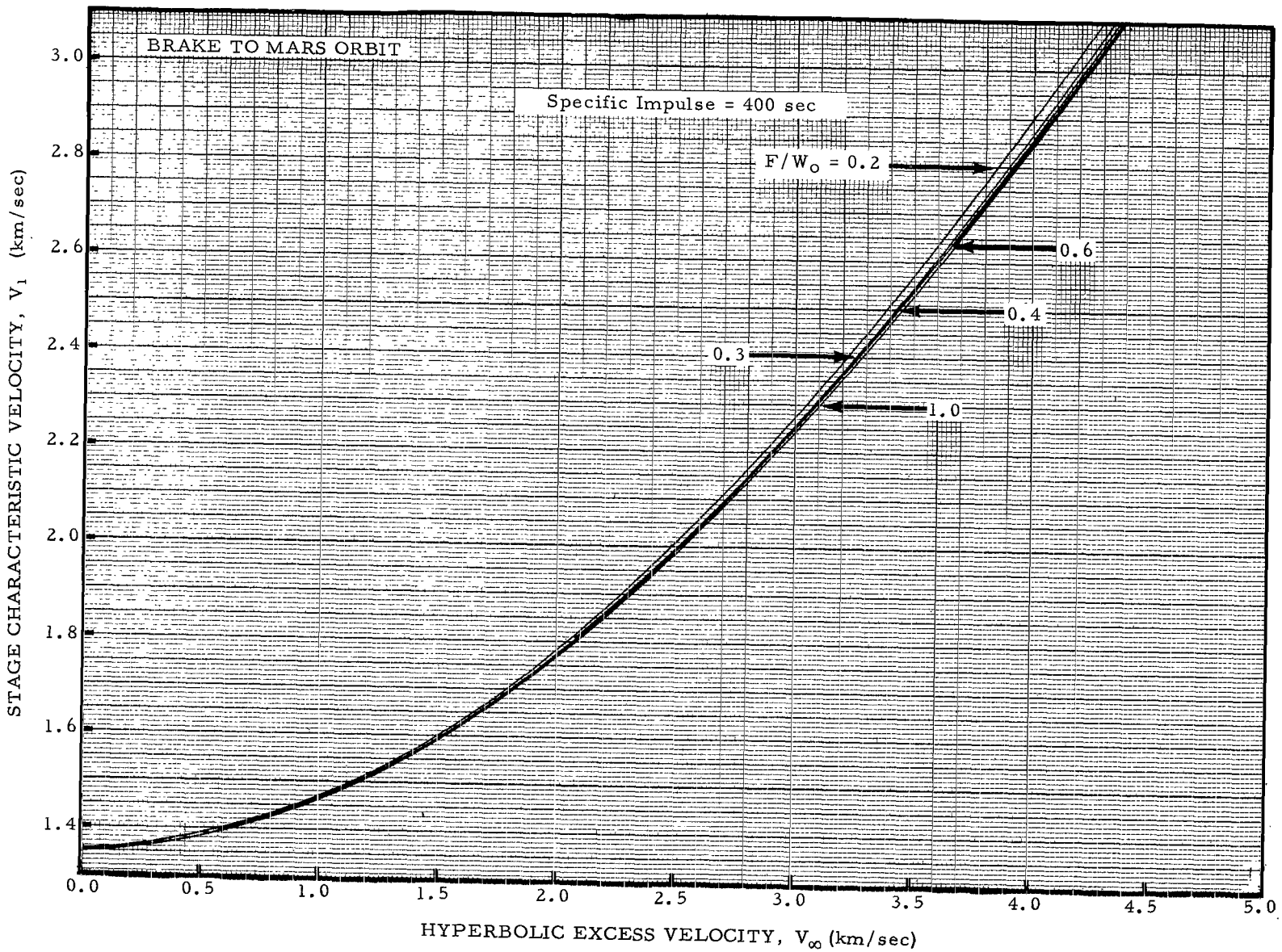


FIGURE 1a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

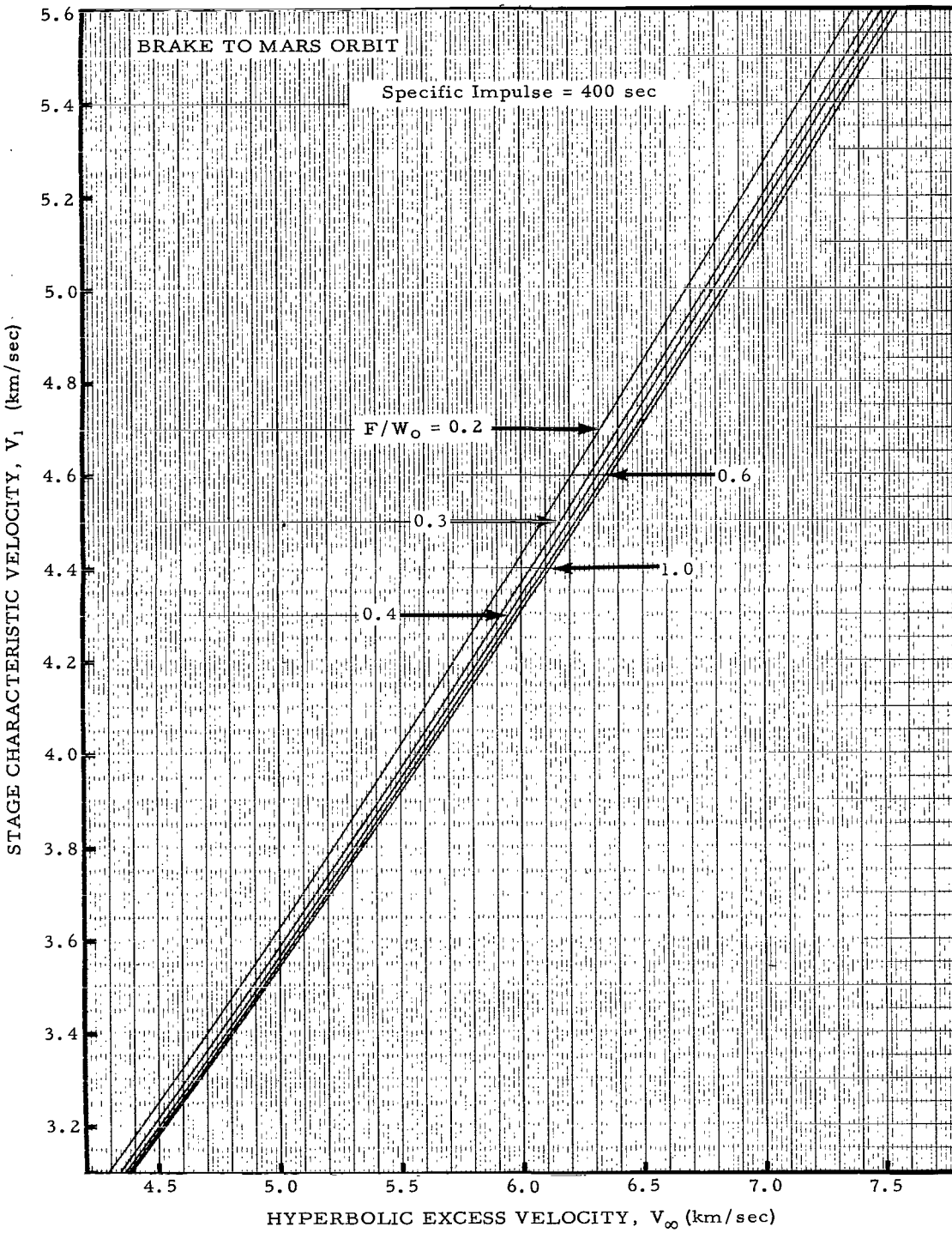


FIGURE 1b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

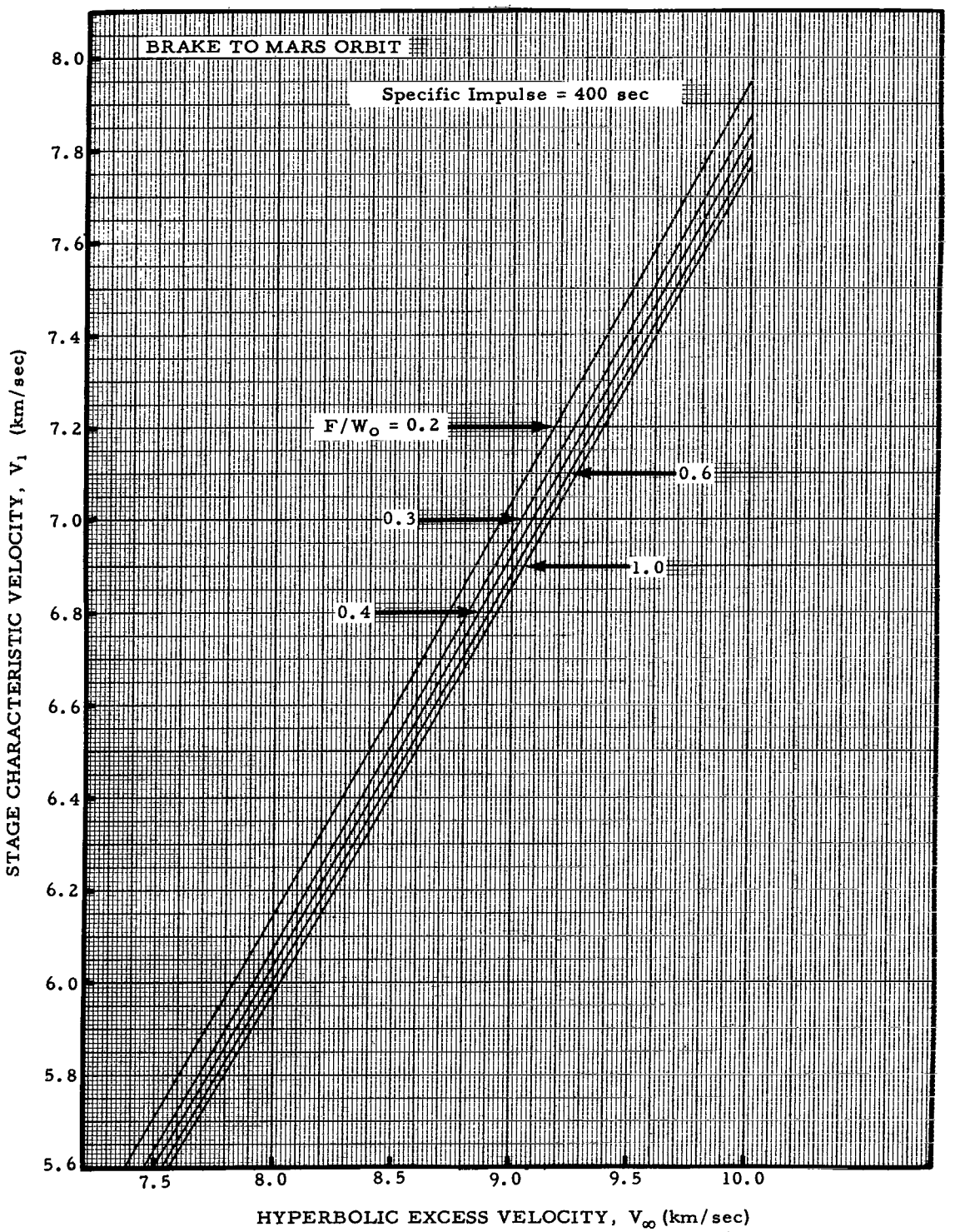


FIGURE 1c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

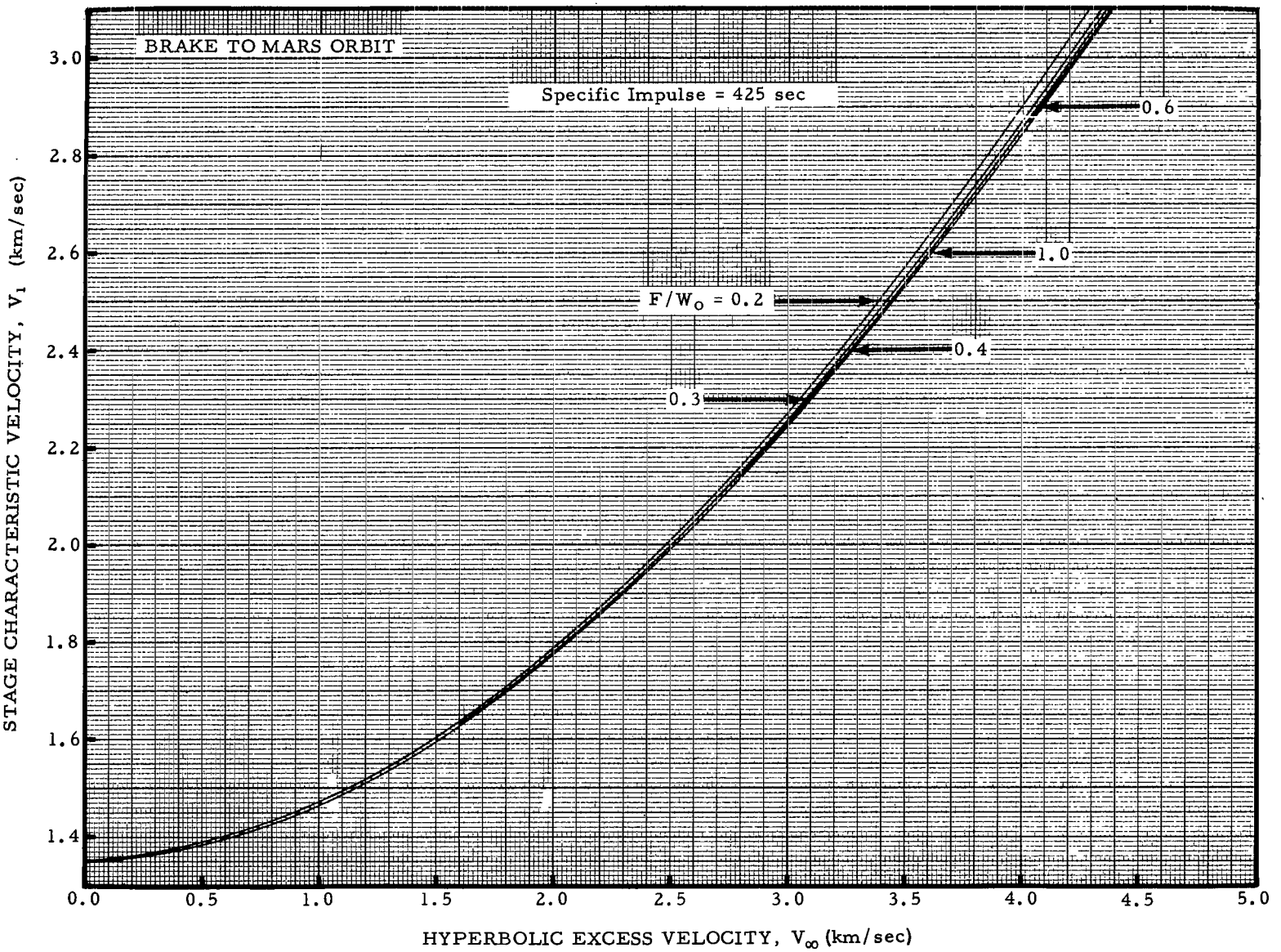


FIGURE 2a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

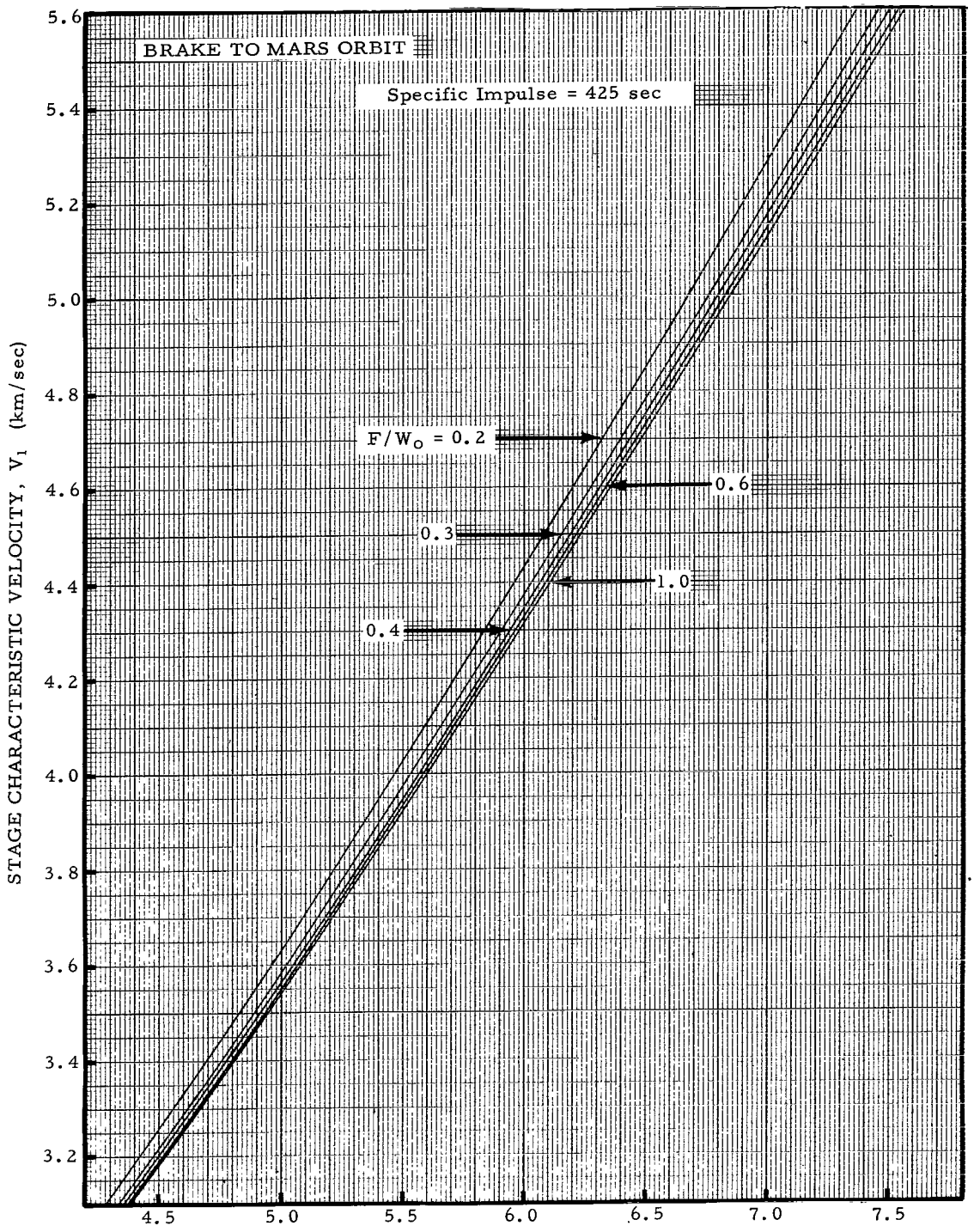


FIGURE 2b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

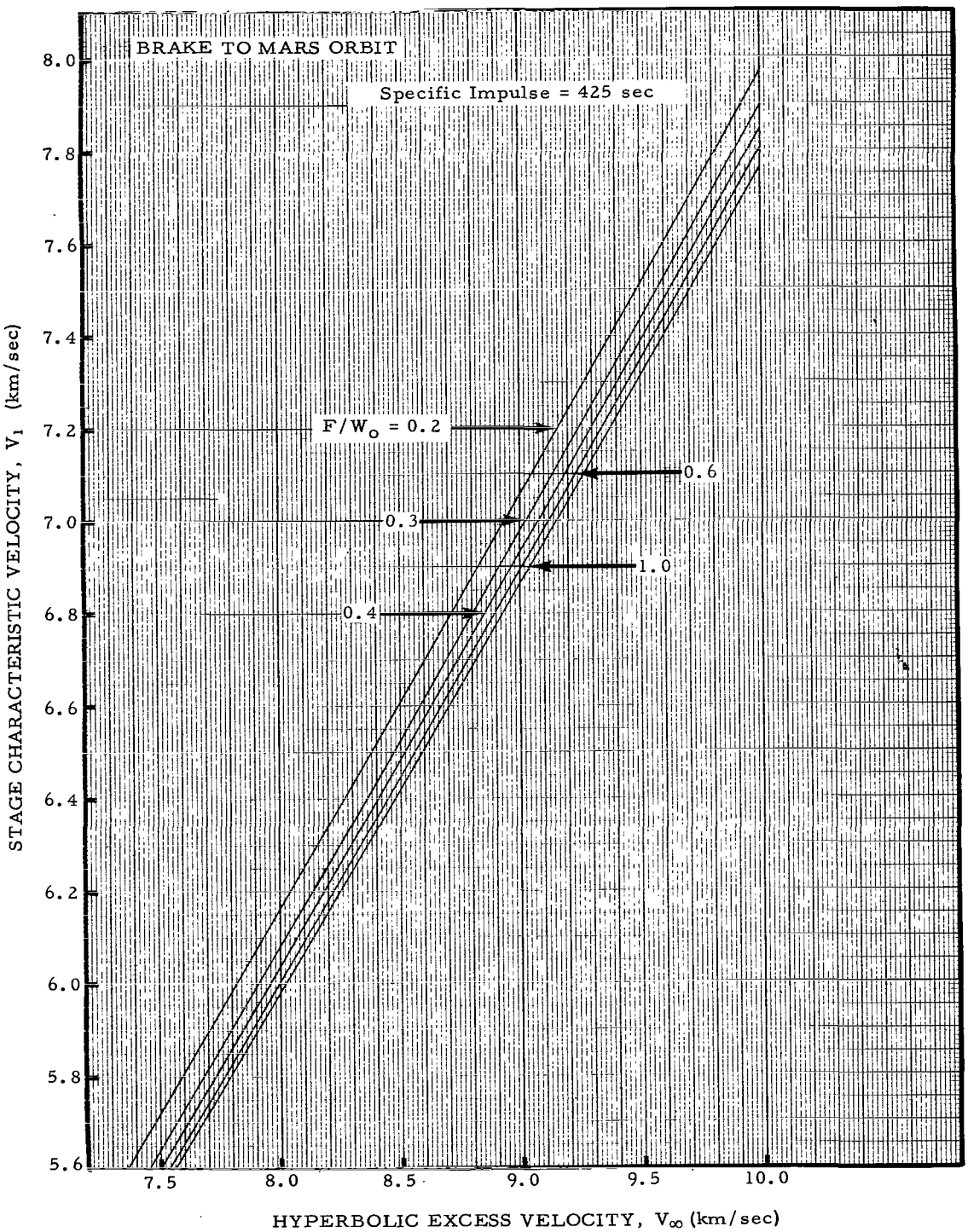


FIGURE 2c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

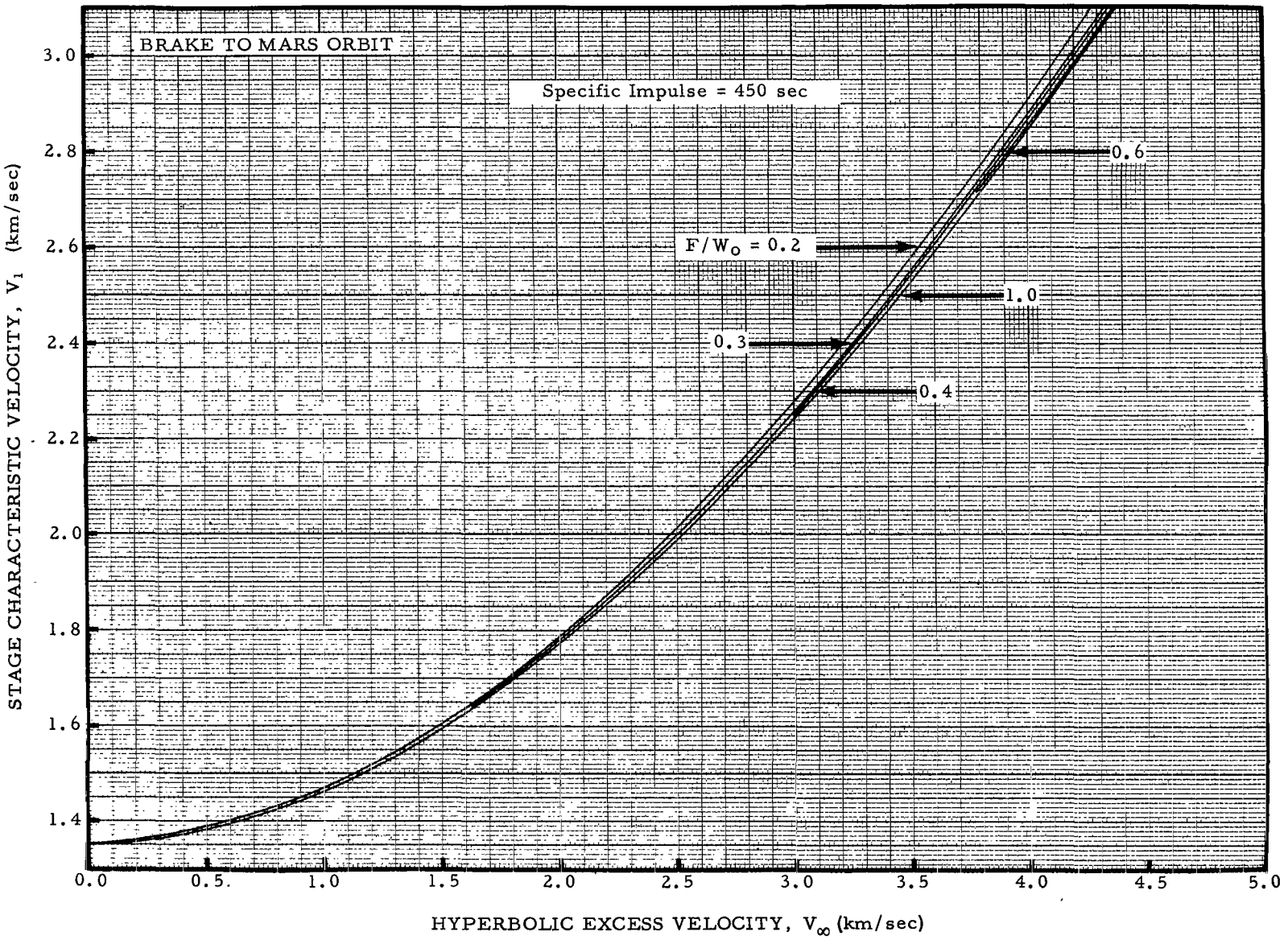


FIGURE 3a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

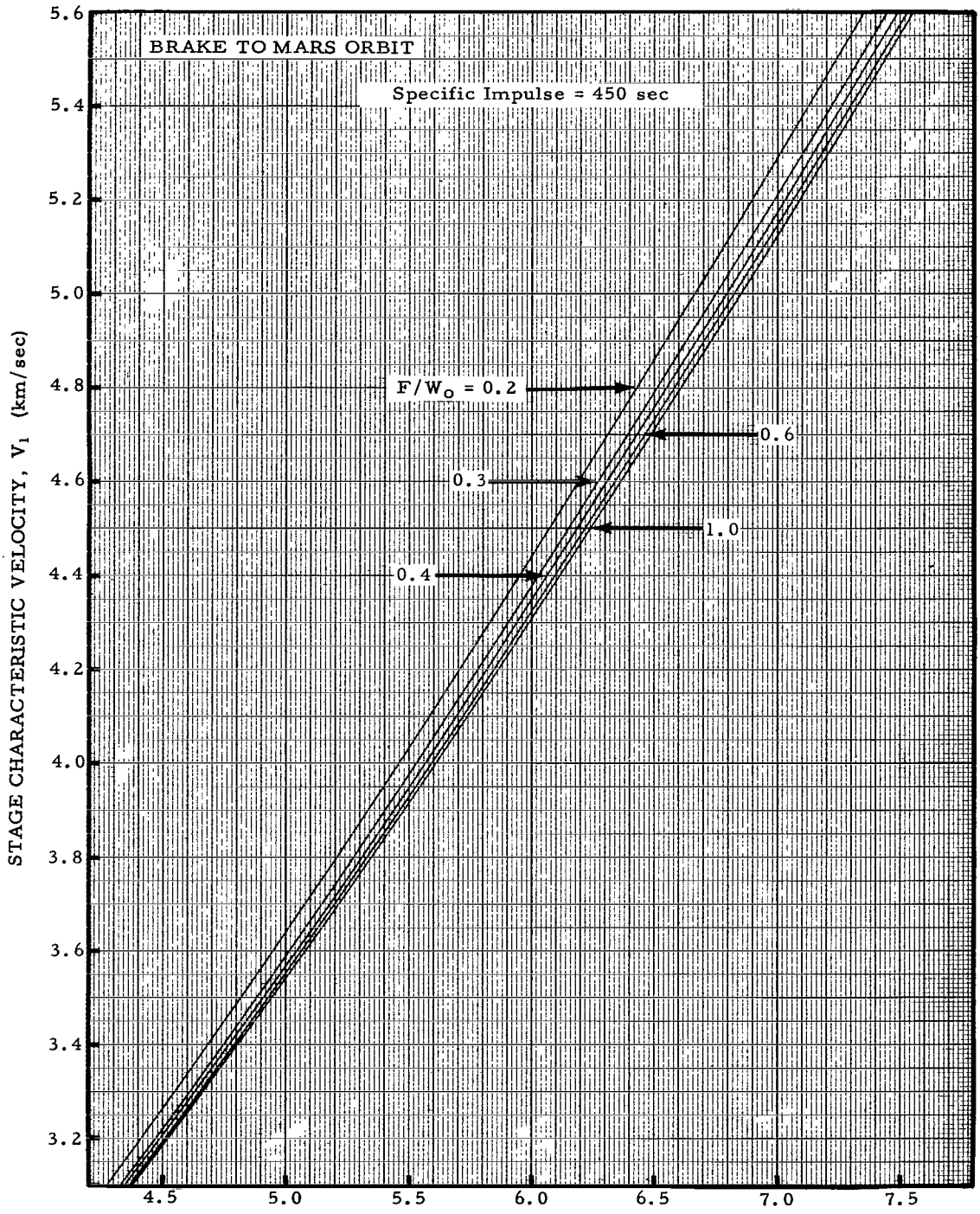


FIGURE 3b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

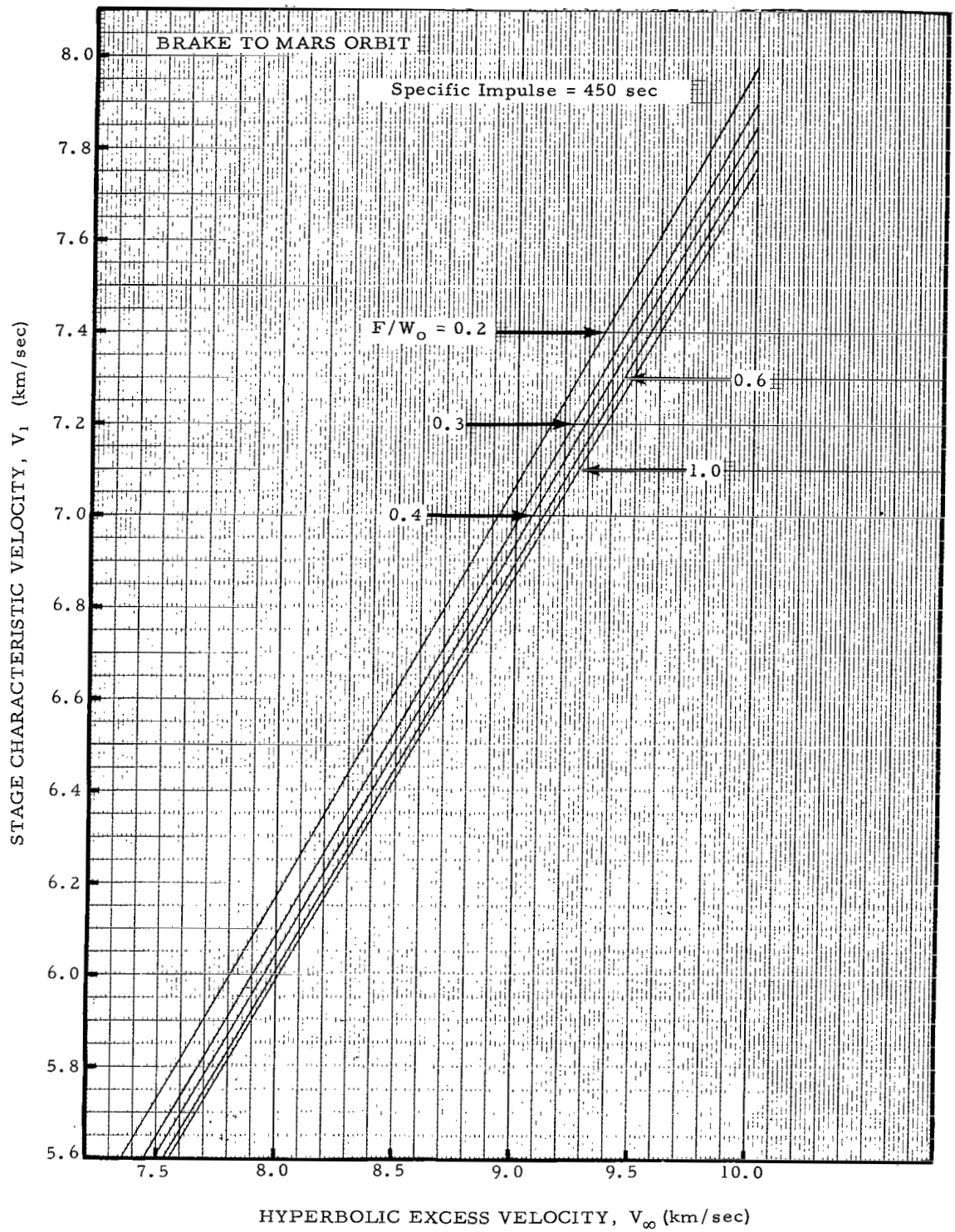


FIGURE 3c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

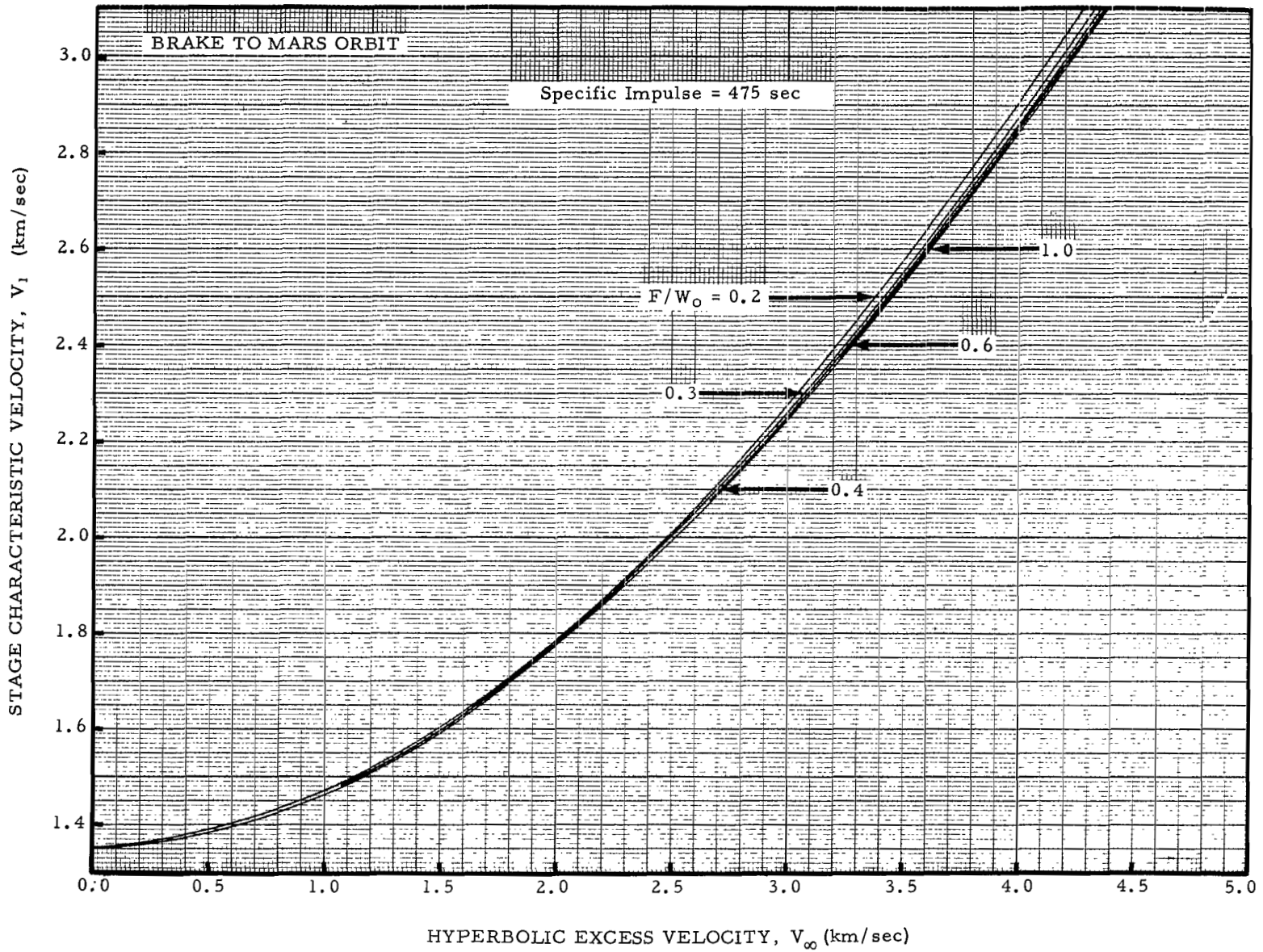
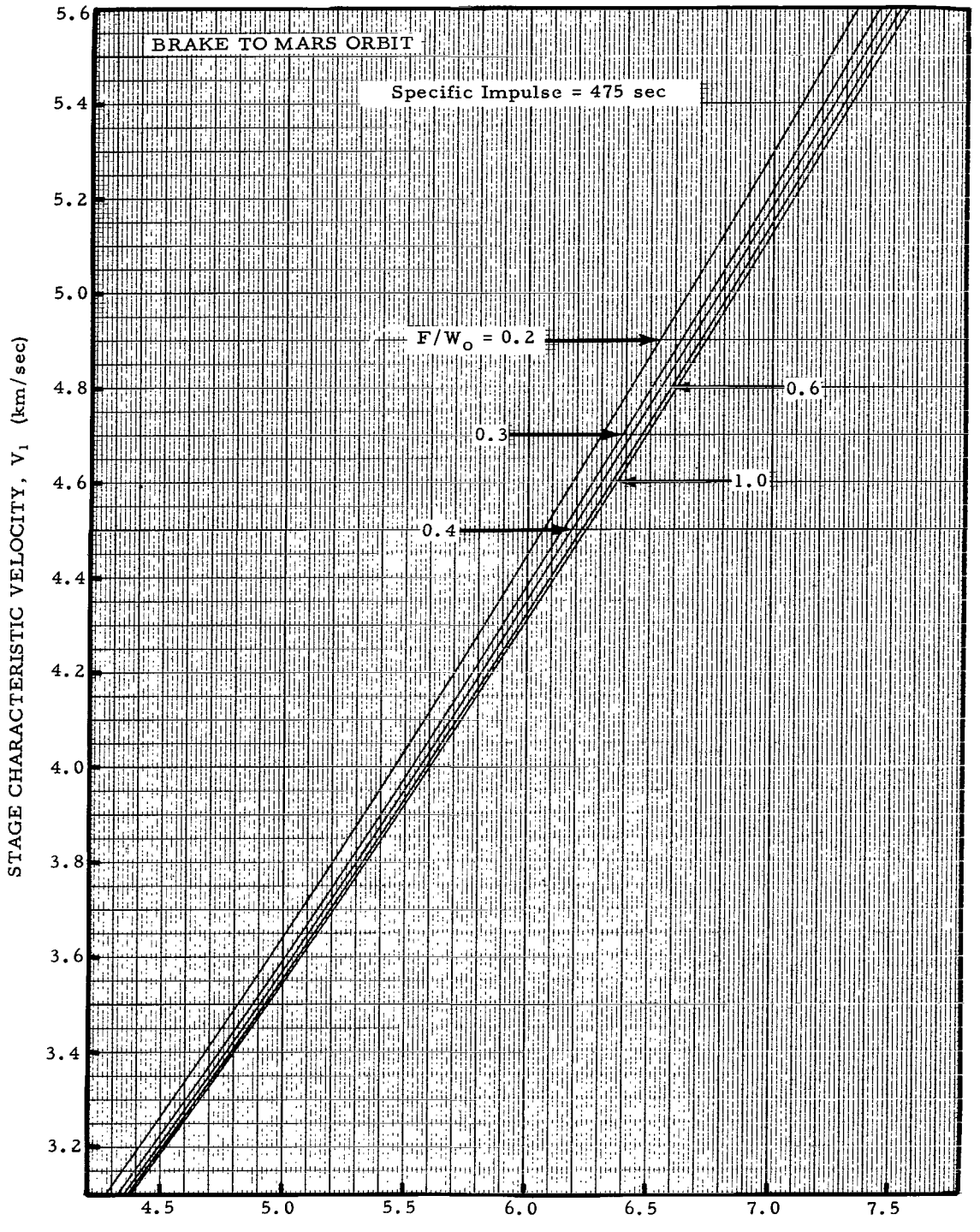


FIGURE 4a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS



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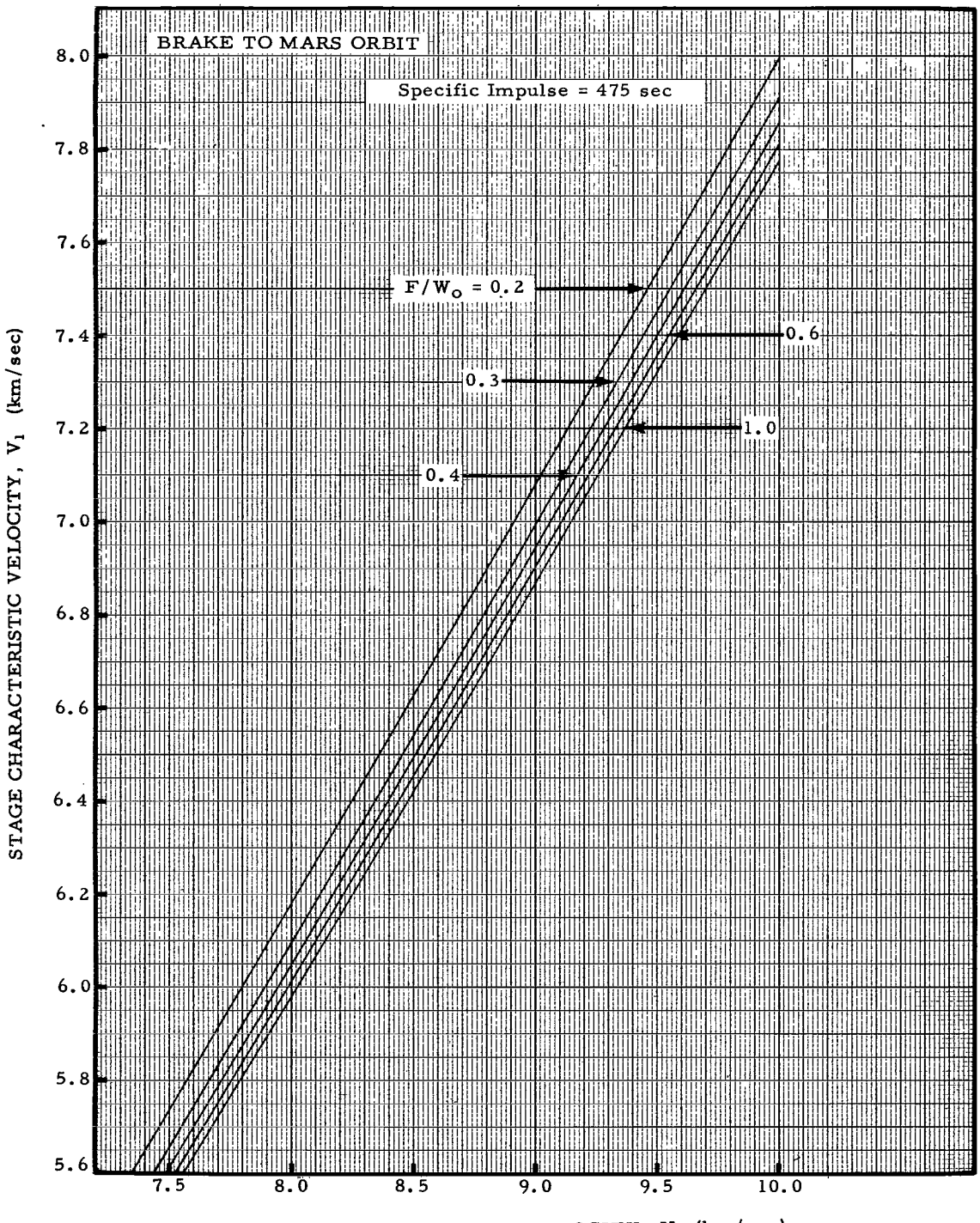

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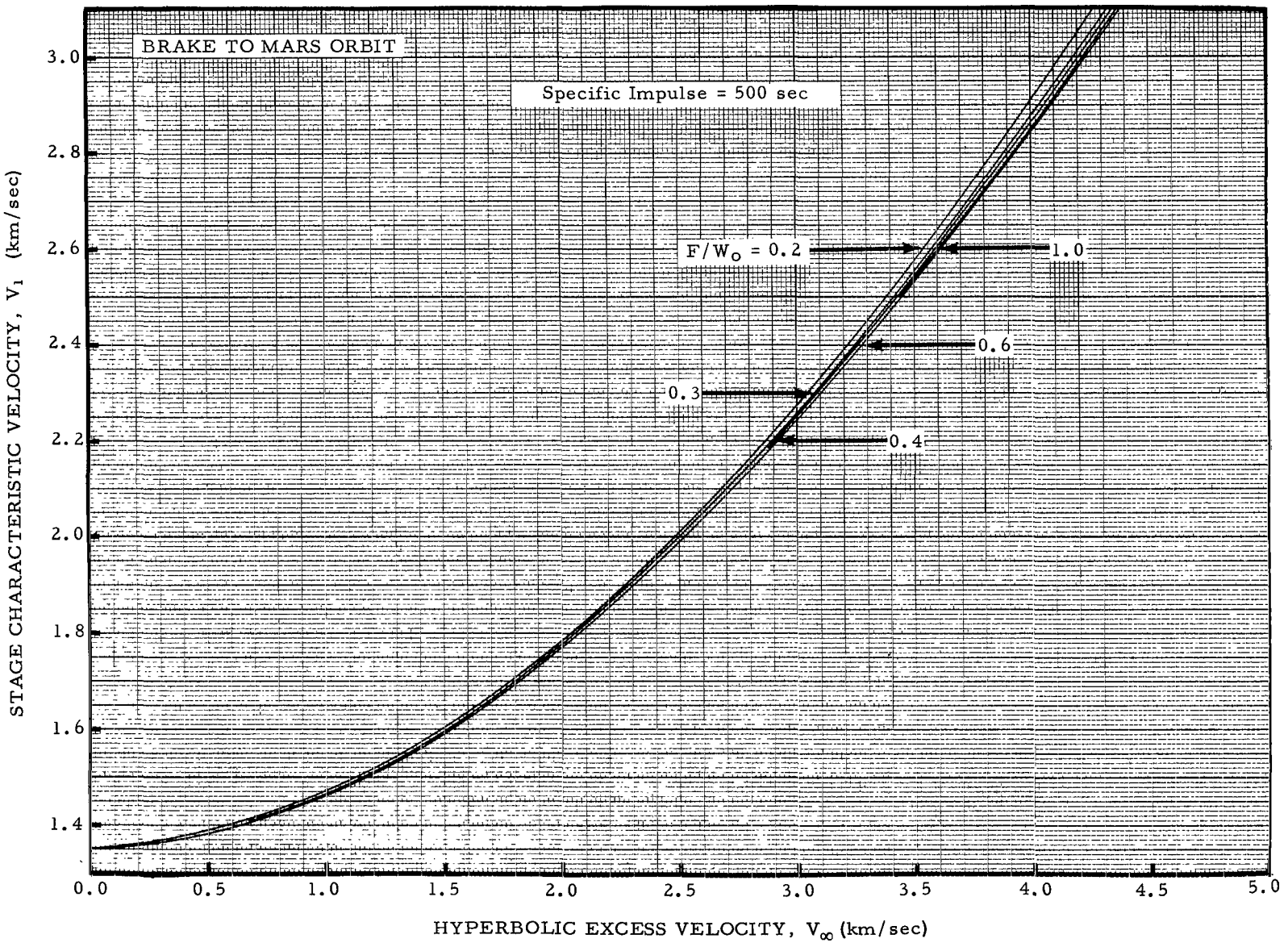

FIGURE 5a. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

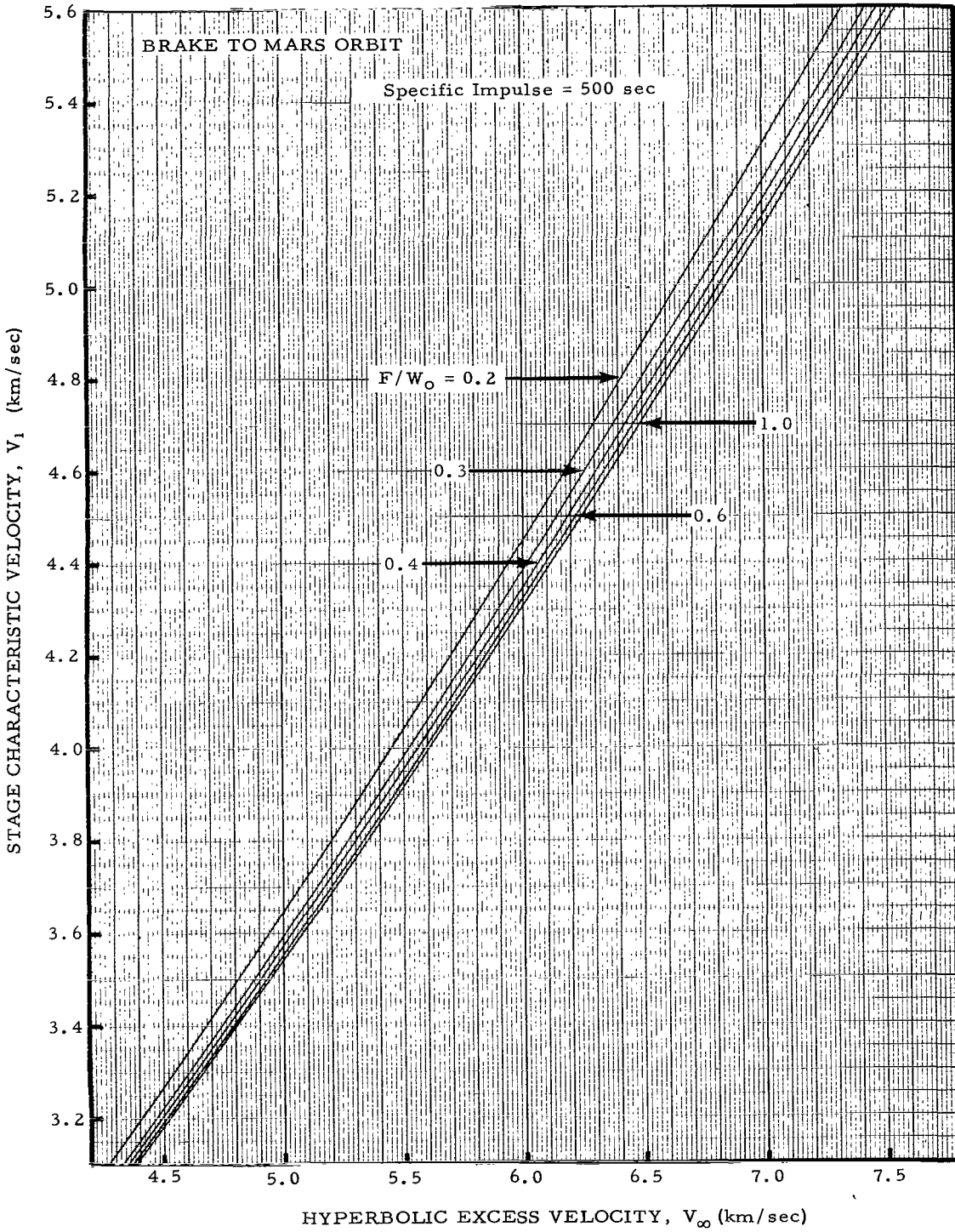

FIGURE 5b. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

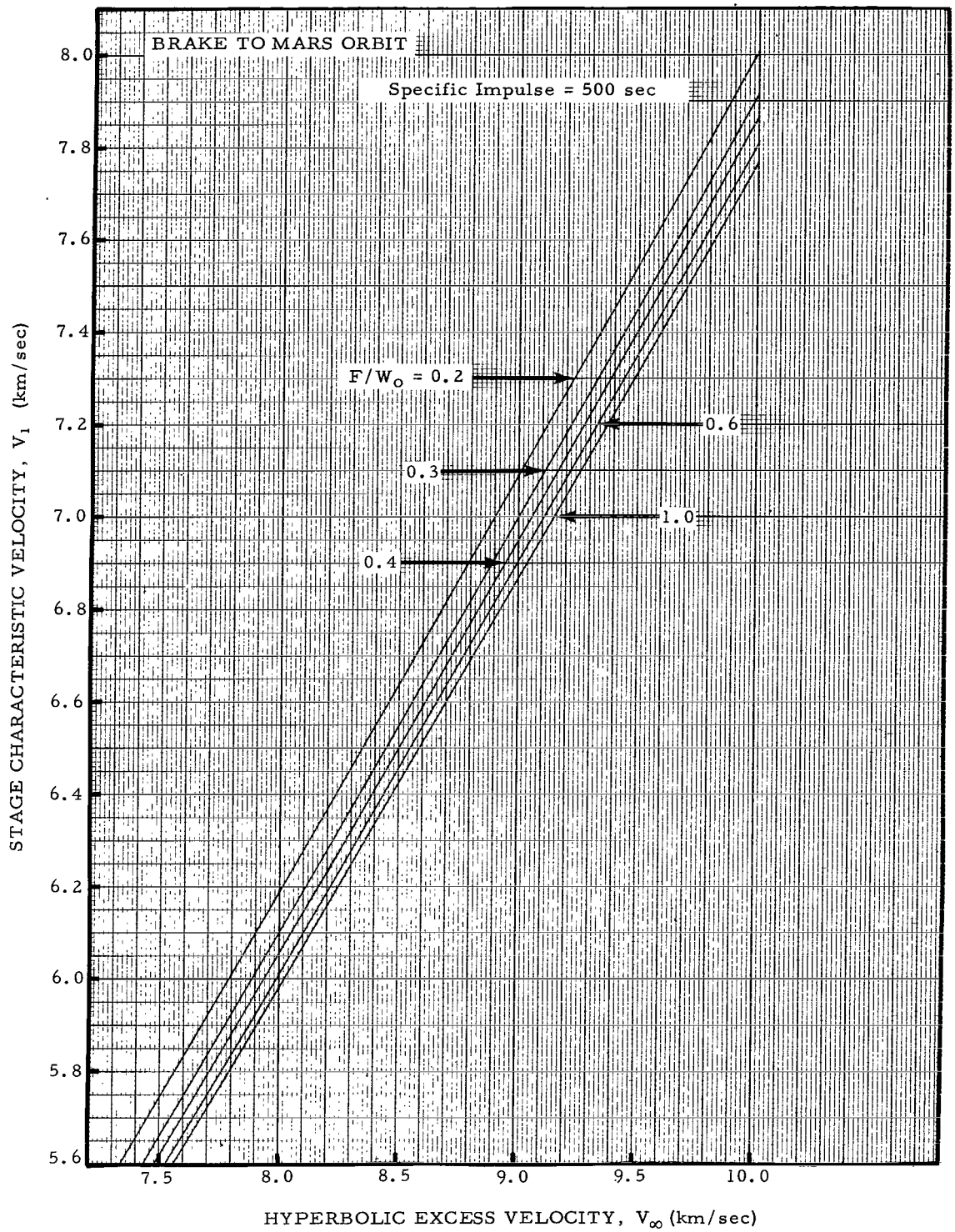

FIGURE 5c. CHARACTERISTIC VELOCITY, V_1 (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

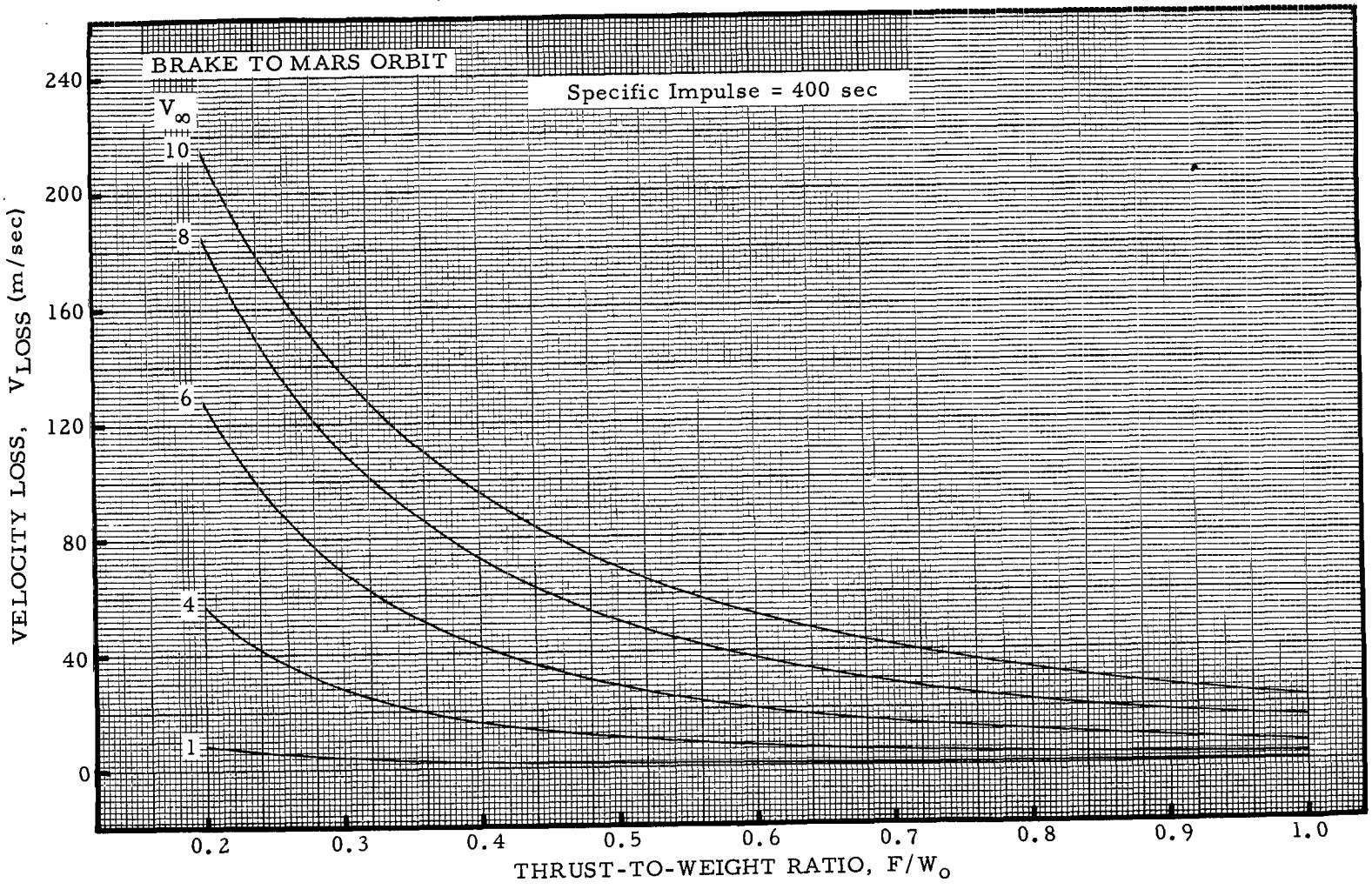

FIGURE 6. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

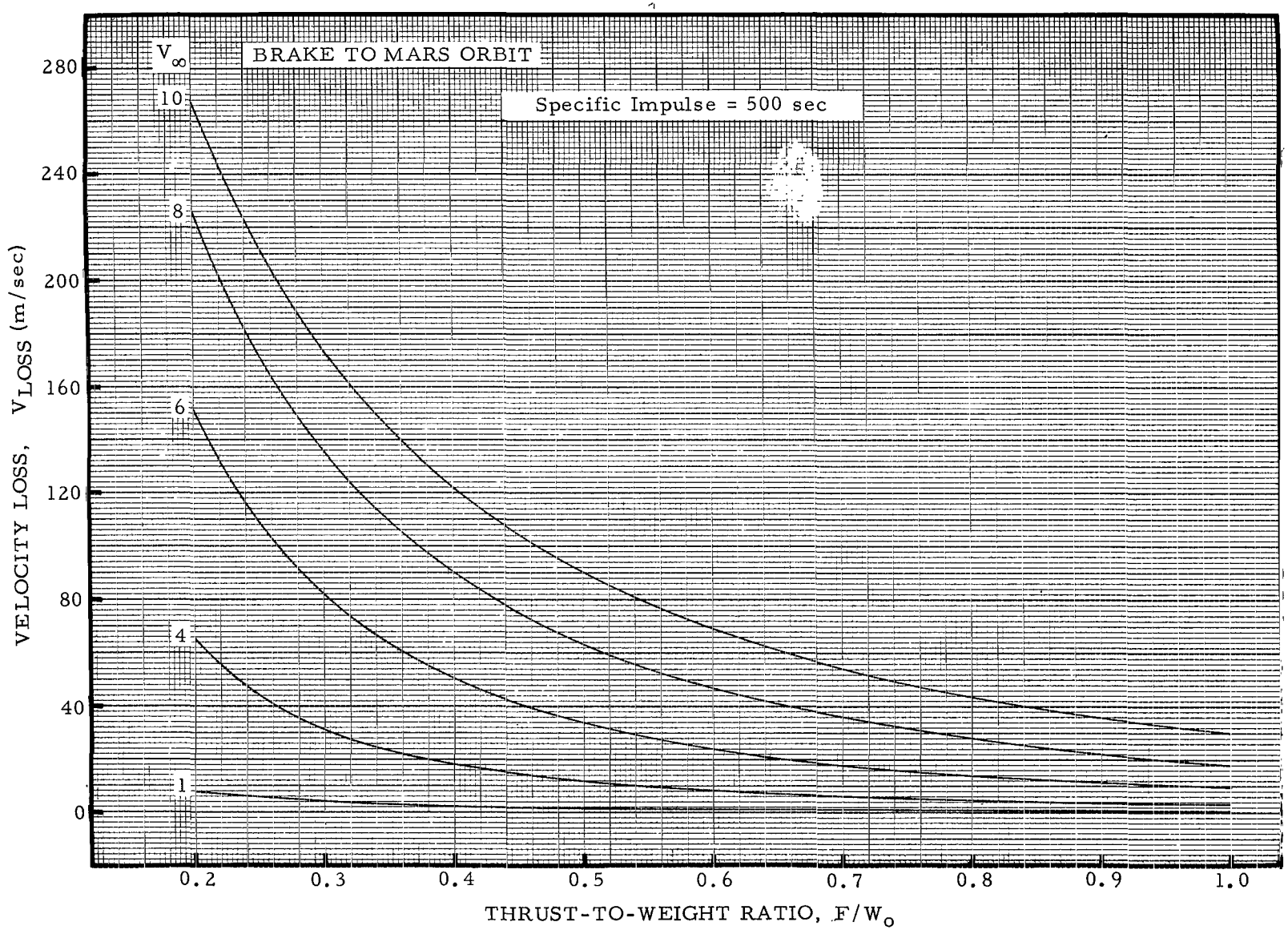

FIGURE 7. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY (km/sec) AS A PARAMETER

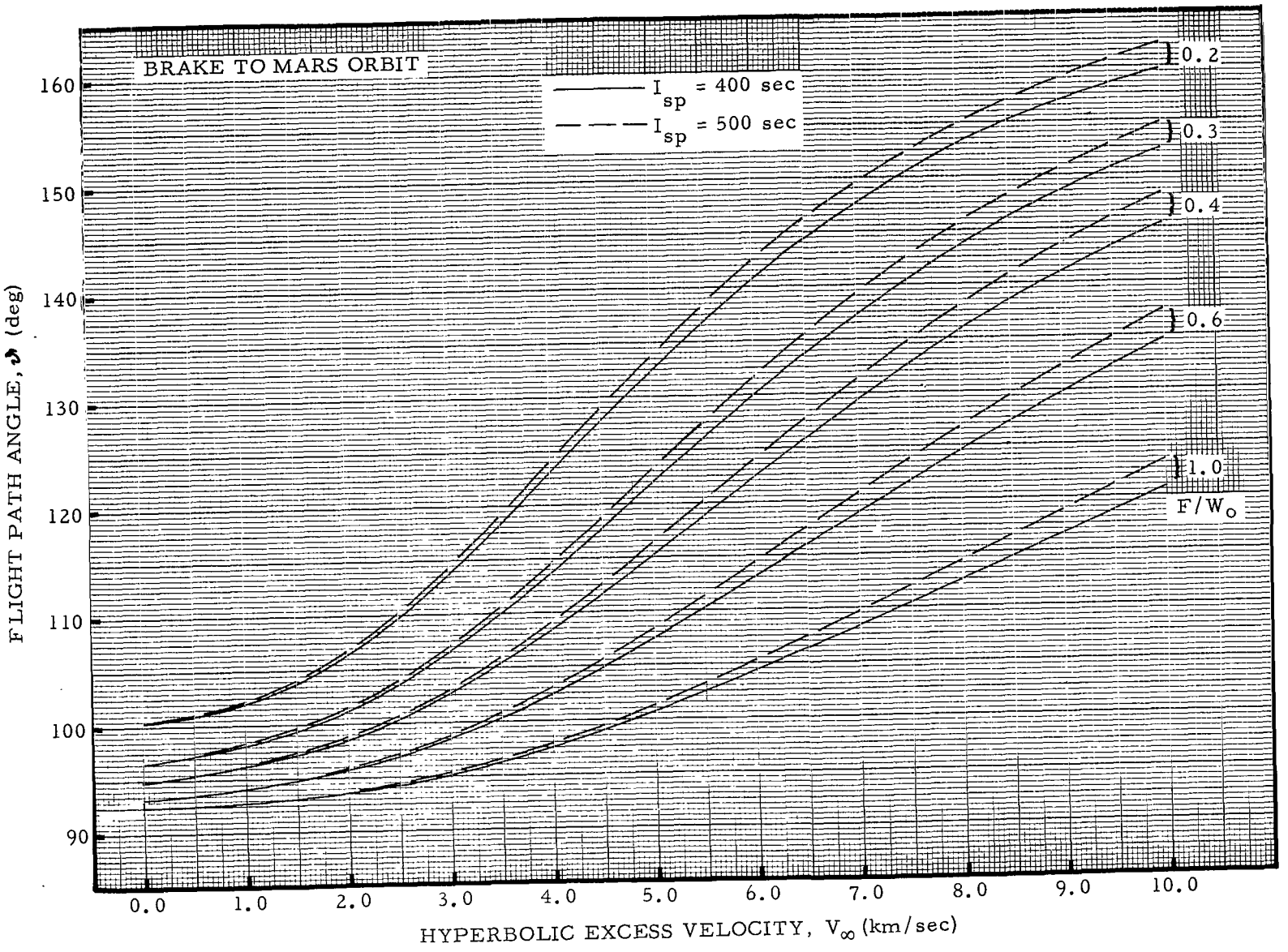

FIGURE 8. FLIGHT PATH ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

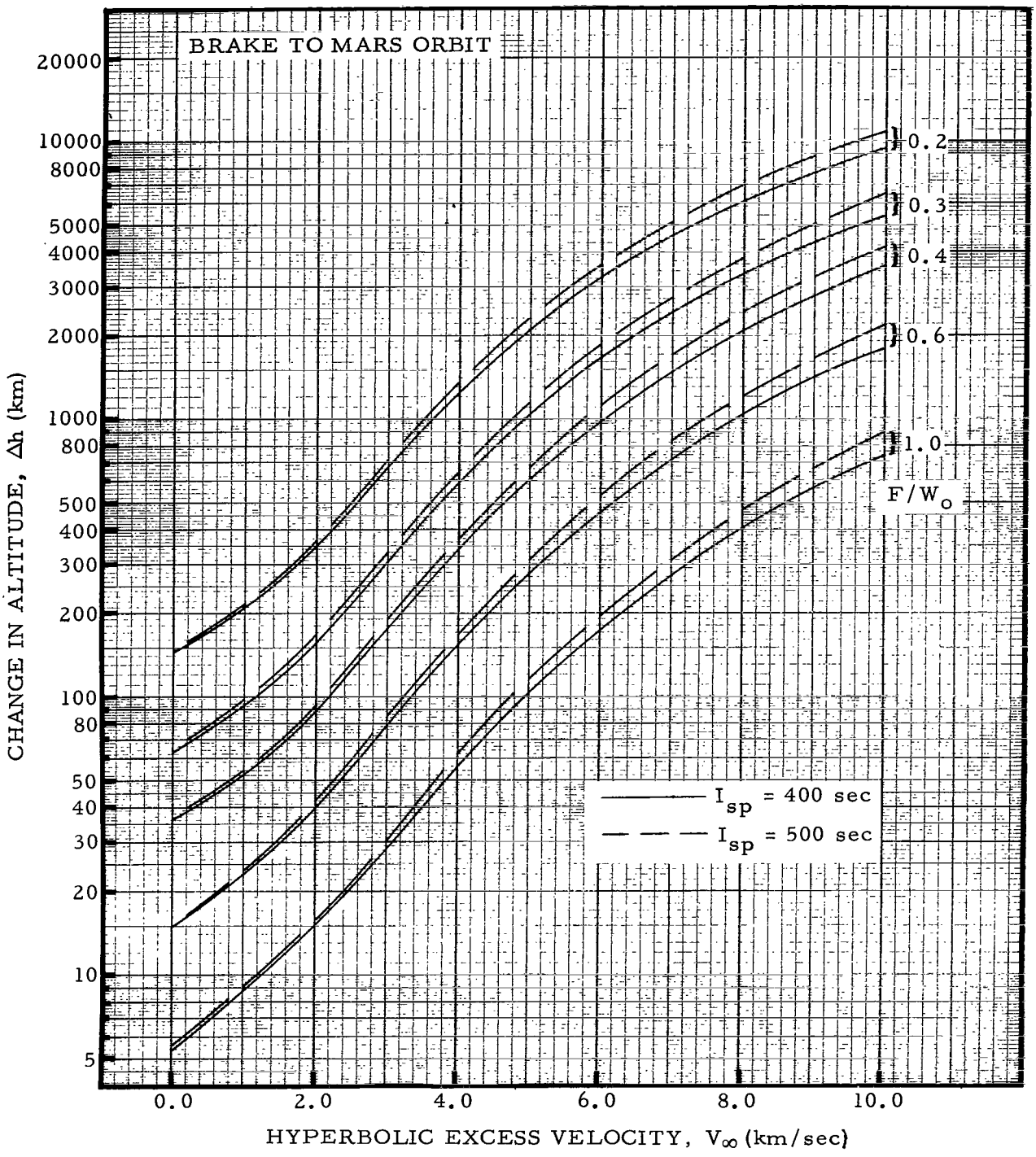

FIGURE 9. CHANGE IN ALTITUDE (km) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

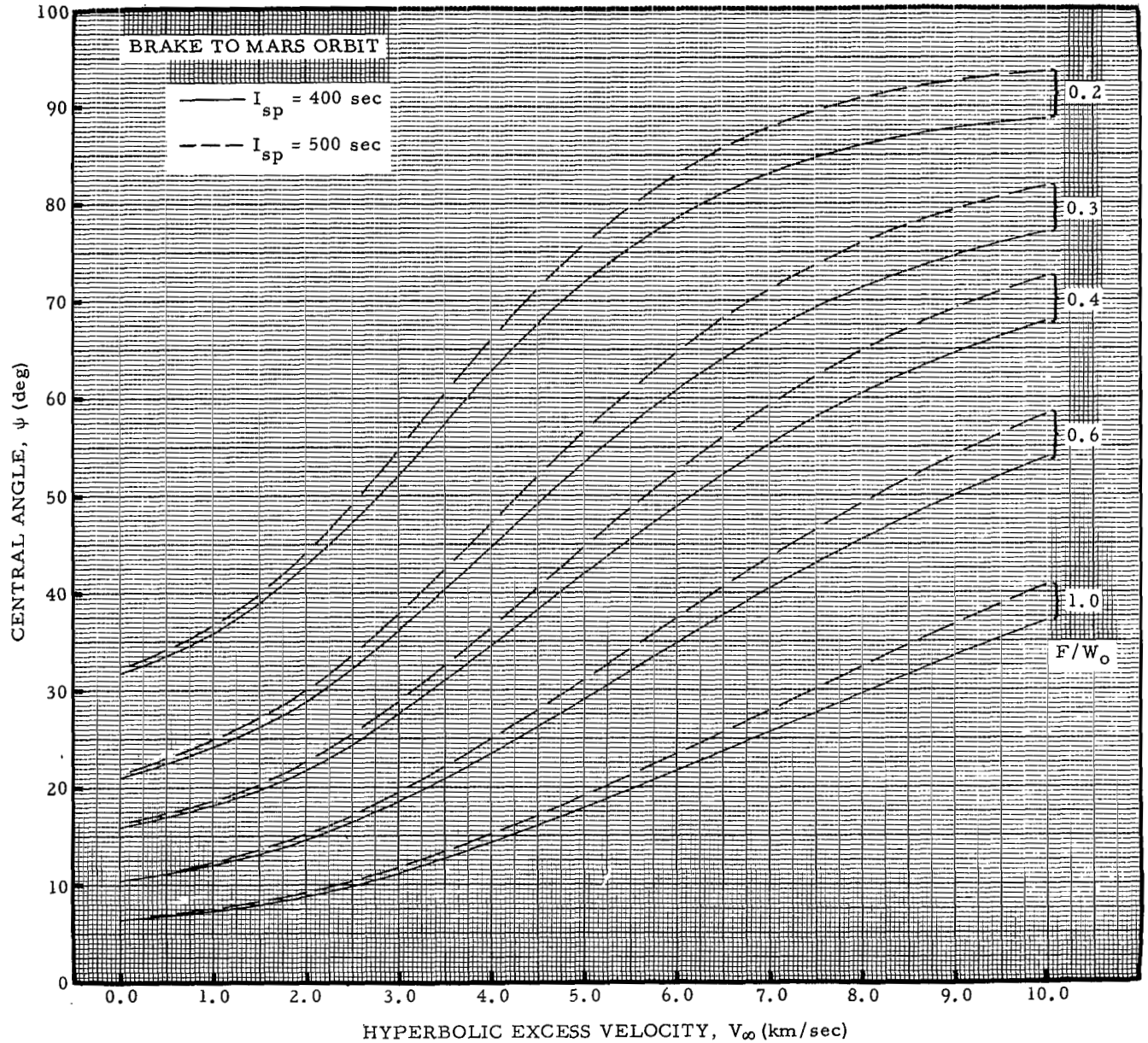

FIGURE 10. CENTRAL ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

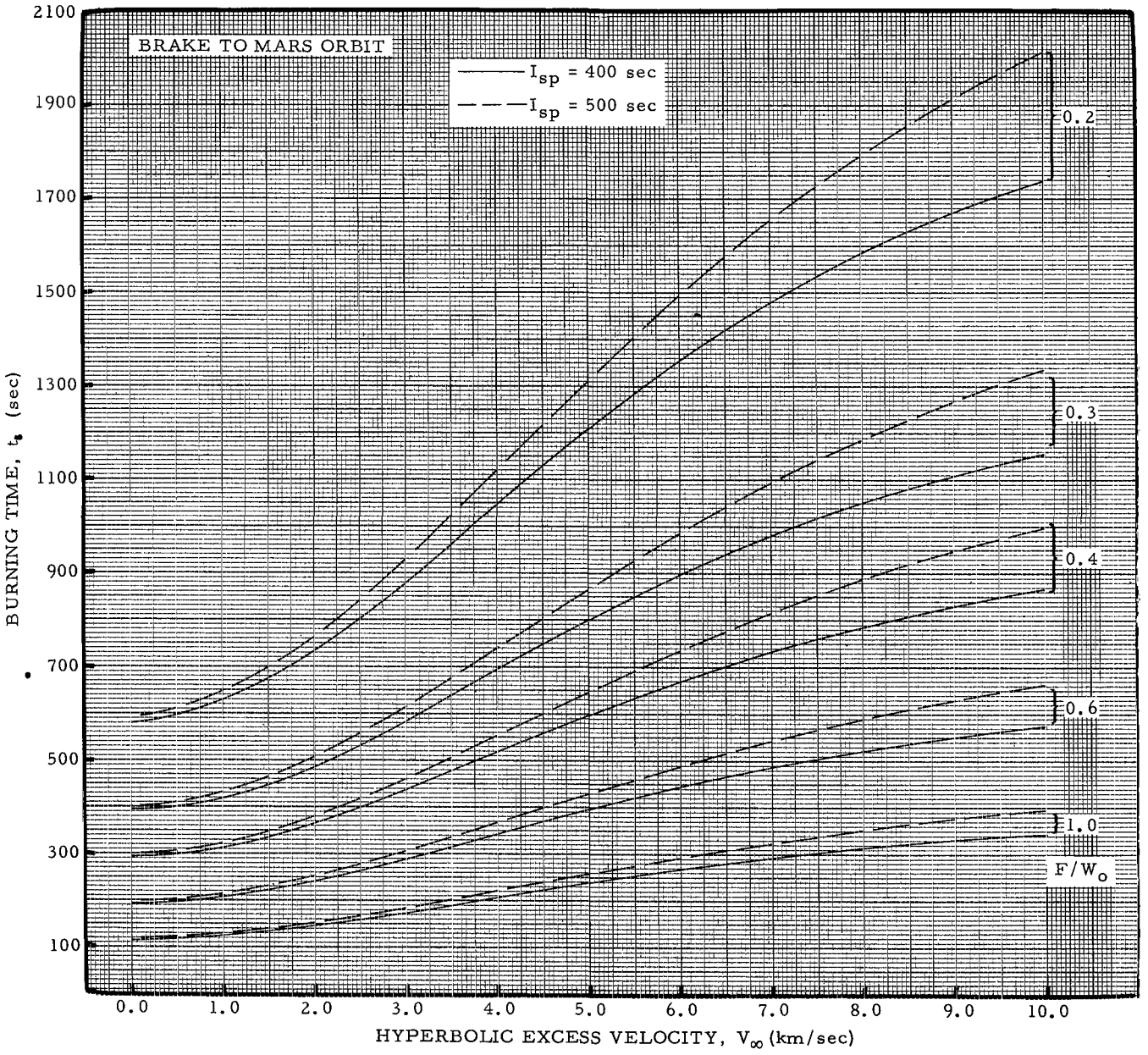

FIGURE 11. BURNING TIME (sec) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

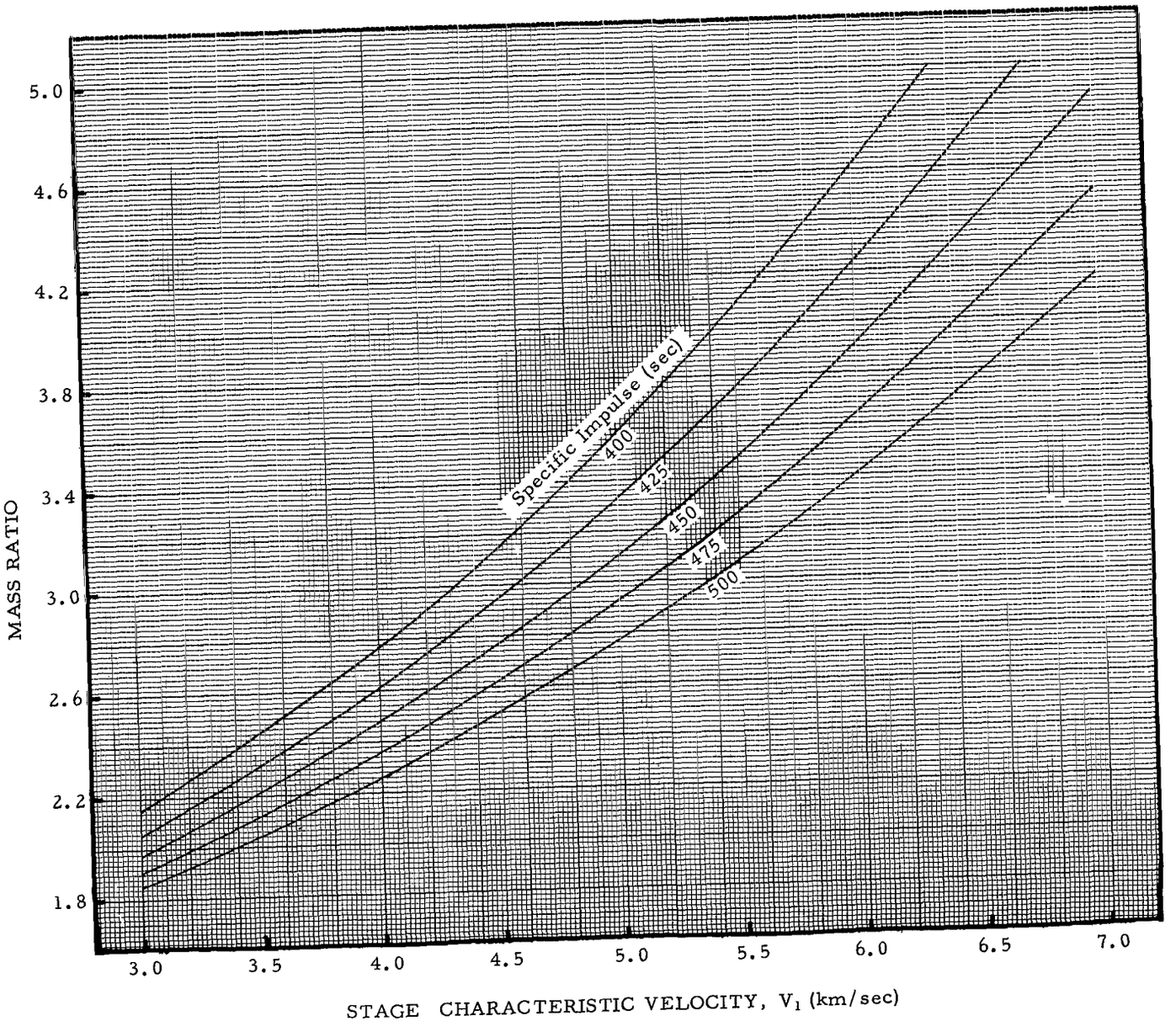

FIGURE 12. MASS RATIO VERSUS CHARACTERISTIC VELOCITY (km/sec) WITH SPECIFIC IMPULSE AS A PARAMETER

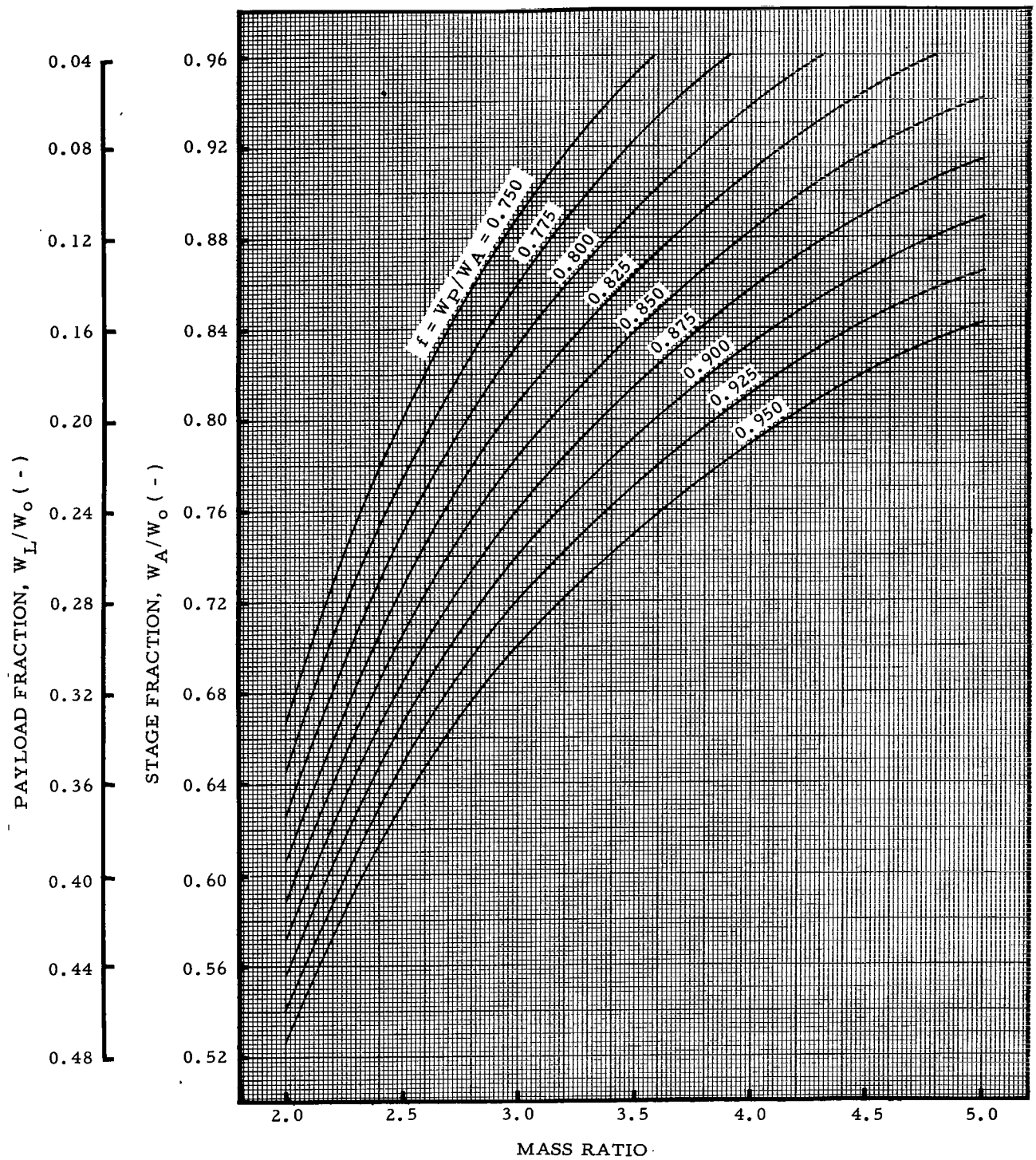

FIGURE 13. PAYLOAD FRACTION AND STAGE FRACTION VERSUS MASS RATIO WITH STAGE PROPELLANT MASS FRACTION AS A PARAMETER

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